The Omega Function
-Not for Circulation-

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1 Introduction

In this paper we introduce a bijection between a class of univariate cumulative distribution functions and a class of monotone decreasing functions which we call Omega functions. More precisely, if F is a univariate cumulative distribution with domain $D = (a, b)$ where a may be $-\infty$ and/or b may be ∞, and F satisfies a simple growth condition,there is a unique monotone function Ω_F from (a, b) to $(0, \infty)$. With very mild assumptions, each such function may be used to reconstruct a unique cumulative distribution. (In fact, there is a closed form expression for the distribution in terms of Ω_F and its first two derivatives [3].)

The correspondence between cumulative distributions and Omega functions is therefore a natural duality. The global properties of the distribution are reflected in local properties of the Omega function. For example, the mean is the unique point at which the Omega function takes the value 1. For a normal distribution with mean μ and variance σ^2, the slope of the Omega function at μ is $\frac{-\sqrt{2\pi}}{\sigma}$. Higher moments are encoded in the shape of the Omega function.

The latter property of the Omega function makes it particularly well suited to the study of statistical data, such as financial time series, in which the deviation from normality is often critical but is difficult to estimate reliably through the use of higher moments due to noisy or sparse data, or is difficult to incorporate into models of markets and behaviour. This application was in fact what led

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to the discovery and development of the Omega function [1, 2]. We present the derivation in its original form here, but note that it may be motivated equally well by any application in which benefits and costs, gains and losses are to be compared.

In section 2 we present the definition of Omega and its most important properties, with proofs of the latter results. In section 3 we provide some examples of Omega functions for standard univariate distributions. Section 4 contains an indication of further results which we will report on separately and a brief discussion of some open questions.

2 The Omega function of a cumulative distribution

We motivate the definition of the Omega function using an example from finance. We consider the returns distribution F for some financial instrument and we let $D{=}(a,b)$ denote the domain of F. Next we set a return level $r{=}L$ in (a,b) which is to be regarded as a loss threshold. In other words, we consider a return above the level L as a gain and a return below L as a loss. With this starting point we now consider the quality of a bet on a gain. To evaluate this, we must consider both the 'odds', namely what we will win if we win and what we will lose if we lose, as well as the 'form' namely the probabilities of winning and losing.

As an example, we might regard the amount by which the conditional expectation $E(r \mid r > L)$ exceeds L as an indicator of what we stand to win if we win. The amount by which the conditional expectation $E(r \mid r < L)$ falls below L is an indicator of what we stand to lose if we lose. If we weight the expected gain $g = E(r - L \mid r > L)$ and loss $l = E(L - r \mid r < L)$ by the respective probabilities of a gain and a loss, namely $1 - F(L)$ and $F(L)$,their ratio, $\frac{E(r-L|r>L)(1-F(L))}{E(L-r|r<L)F(L)}$ is a rough measure of the quality of our bet.

Of course the conditional expected gain and loss are only one pair of infinitely many possible gains and losses. For example we may consider separately the possibility of the gains from returns of $L + \Delta x$ and $L + 2\Delta x$ with probability weights of $1-F(L)$ and $1-F(L+\Delta x)$ respectively. We also have the possibility of losses from returns of $L - \Delta x$ and $L - 2\Delta x$ with probability weights of $F(L)$ and $F(L - \Delta x)$ respectively. Taking these possible returns into account leads to $\frac{(1-F(L))\Delta x+(1-F(L+\Delta x))\Delta x}{F(L)\Delta x+F(L-\Delta x)\Delta x}$ as an estimate of the quality of a bet on a return above the level of L. If we continue in this vein, letting our unit of gain and loss, Δx, tend to zero, we are led, in the limit, to the the integral

$$I_2(L) = \int_L^b 1 - F(x)dx. \tag{1}$$

as the total probability weighted gains and to the integral

$$I_1(L) = \int_a^L F(x)dx \tag{2}$$

as the total probability weighted losses. The ratio of these is our measure of probability weighted gains to losses for a bet on a return above $r = L$ (assuming, as we do henceforward, that these integrals are finite.)

If we now let the risk threshold L run over the domain of the cumulative distribution of returns, we obtain the function Ω_F , which we call the Omega function of the distribution F, defined on (a, b) by

$$\Omega_F(r) = \frac{I_2(r)}{I_1(r)}. \tag{3}$$

where

$$I_1(r) = \int_a^r F(x)dx \tag{4}$$

and

$$I_2(r) = \int_r^b 1 - F(x)dx. \tag{5}$$

It is obvious that each cumulative distribution leads unambiguously to an Omega function, provided that the integrals in question exist for finite values of r. Among the properties of this function which we extablish in the theorem, is the fact that the map from distributions to Omega functions is one to one. (The inverse problem of reconstructing a distribution from an Omega function is the subject of a separate article [3].)

Theorem 1 *If F and G are continuous distributions on (a,b) such that the integrals (4) and (5) exist for all r in (a,b) then*

1) $\Omega_F(r) = \frac{E_F(C(r))}{E_F(P(r))}$ where $C(r)$=$max(x-r,0)$, $P(r) = max(r-x,0)$ and E_F denotes expectation with respect to the distribution F.

2)Ω_F is differentiable and $\frac{d\Omega_F}{dr} = \frac{F(r)(I_1(r)-I_2(r))}{I_1^2(r)} - \frac{1}{I_1(r)}$

3)Ω_F is a monotone decreasing function from (a,b) to $(0,\infty)$.

4) If ϕ is an affine diffeomorphism of (a,b), let H denote the induced distribution,$H = F\circ\phi^{-1}$. If $\frac{d\phi}{dr} > 0$ then $\Omega_H(\phi(r))$=$\Omega_F(r)$ for all r in (a,b). If $\frac{d\phi}{dr} < 0$ then $\Omega_H(\phi(r))$=$\Omega_F(r)^{-1}$ for all r in (a,b).

5) If μ is the mean of the distribution F then $\Omega_F(\mu)$=1.

6) F is symmetric about μ if and only if $\Omega_F(r-\mu)$=$\Omega_F(r+\mu)^{-1}$ for all r in (a,b).

7) If F_λ is a smooth 1-parameter family of deformations of F with $F_0 := F$ and $F^{(1)} = \frac{dF_\lambda}{d\lambda}\mid_{\lambda=0}$, let Ω_λ denote Ω_{F_λ}. Then
$\frac{d\Omega_\lambda}{d\lambda}\mid_{\lambda=0} = \frac{1}{I_1(r)^2}(-\int_a^r F^{(1)}(x)dxI_2(r) - \int_r^b F^{(1)}(x)dxI_1(r))$.

8) $\Omega_F = \Omega_G$ if and only if $F = G$.

Proof.

1) This follows immediately from the definition of Ω_F and the fact that $E_F(\phi(x)) = \int_0^b 1 - F_{\phi(x)}(y)dy - \int_a^0 F_{\phi(x)}(y)dy$ where $F_{\phi(x)}$ is the distribution induced by ϕ.

2) This follows directly from the definition of Ω_F.

3) This follows from 2) and the definition of Ω_F.

4) This follows from the definition of Ω_F by making a change of variable in the integrals $I_1(r)$ and $I_2(r)$.

5) If $\mu = 0$ it is immediate from the definitions of $I_1(r)$ and $I_2(r)$ that $I_2(0) - I_1(0) = 0$ which establishes 5) in this special case. The general case, for a distribution F with $\mu \neq 0$ now follows by using the translation $\phi(x) = x - \mu$. The distribution $H = F\circ\phi^{-1}$ induced by ϕ has its mean at 0 so $\Omega_H(0) = 1$. But $\Omega_H(0) = \Omega_F(\mu)$ by 4).

6) This follows from the definition of Ω_F by changing variables in $I_1(r)$ and $I_2(r)$.

7) This follows from the definition of Ω_{F_λ} by exchanging the orders of differentiation and integration.

8) It is immediate that when $F = G$, $\Omega_F = \Omega_G$. Next we assume that $\Omega_F = \Omega_G$. Let $\Omega_F(r) = \frac{I_2(r)}{I_1(r)}$ as usual and let $\Omega_G(r) = \frac{J_2(r)}{J_1(r)}$. We note first that it follows from $\Omega_F = \Omega_G$ that $\mu_F = \mu_G$ as the mean is the inverse image of 1. We use μ to denote the common mean and will show that for all $r > 0$, we have $I_1(\mu - r) = J_1(\mu - r)$. Let $I := I_1(\mu) = I_2(\mu)$ and $J := J_1(\mu) = J_2(\mu)$. If we let $A_F = \int_{\mu-r}^{\mu} 1 - F(x)dx$ and $B_F = \int_{\mu-r}^{\mu} F(x)dx$, then $I_2(\mu - r) = I + A_F$ and $I_1(\mu - r) = I - B_F$. With the obvious notation for A_G and B_G we also have $J_2(\mu - r) = J + A_G$ and $J_1(\mu - r) = J - B_G$. Since $A_F + B_F = r$ and $A_G + B_G = r$, the equality of Ω_F and Ω_G at $\mu - r$ gives $\frac{J+r-B_G}{J-B_G} = \frac{I+r-B_F}{I-B_F}$. It follows that $r(I - B_F) = r(J - B_G)$ and, as $r \neq 0$,$I - B_F = J - B_G$. But this is the same as $I_1(\mu - r) = J_1(\mu - r)$ and this relation therefore holds for all $r > 0$. The same argument shows that this holds for all $r < 0$ as well and, by continuity, it holds at $r = 0$ and hence we have $I_1(r) = J_1(r)$ for all r. By differentiating this relation we obtain $F = G$. ■

3 The Omega functions for some standard univariate distributions

The Omega function is a non-local function of a distribution. This means that it encodes some properties of the distribution in a way which is not immediately obvious from the appearance of the graph of either the distribution or its density. This information lends itself naturally to graphical interpretation. For example, if we consider two symmetric distributions with the same mean, the one with fatter tails will have a flatter Omega function asymptotically to the right and left of the mean. Because they must agree at the mean, it follows that this will lead to an odd number of crossings of the Omega functions and thus to some significant changes in shape, even though the Omega function is monotone decreasing.

In this section, we use the Omega functions for some standard distributions to illustrate some of these features. Because of the rapid growth and decay of these functions, and in order to make some of their symmetries more apparent, it is convenient to present graphs of the logarithms of the Omega functions. Likewise, we use the logarithmic derivative to illustrate the changes in shapes.

We begin with the Omega function for the normal distribution. It is easy to verify from the definition of Ω that for a normal distribution with mean μ and standard deviation σ, the slope of Ω at μ is $-\frac{\sqrt{2\pi}}{\sigma}$.

Figure 1 illustrates the natural logarithm of the Omega function for three normal distributions with a common mean of 0 and $\sigma = 1, \sigma = 0.75$ and $\sigma = 0.5$ respectively. The derivatives of $\log(\Omega)$ for the same values of σ are shown in the bottom panel.

Next we illustrate the Omegas for the logistic distribution whose density is $L(r, a, b) = \frac{\exp(\frac{-(r-a)}{b})}{b(1+\exp(\frac{-(r-a)}{b}))^2}$. Figure 2 shows the natural logarithms of the Omegas for $L(r, 0, 0.5)$, $L(r, 0, 0.6)$ and $L(r, 0, 1)$ together with the derivatives of $\log \Omega$. The logistic distribution has fatter tails than a normal with the same mean and variance. Figure 3 contrasts the logarithms of the Omegas for a logistic and a normal distribution with the same mean and variance. Because of its fatter tails, the Omega function for the logistic distribution is dominated by that of the normal as r tends to $-\infty$ and vice versa as r tends to ∞. This produces three crossings which are just discernable in Figure 3. In spite of the close agreement of the two Omega functions near the mean, their shapes are quite different. The bottom panel shows the extent of the differences in shape of the two Omega functions.

Now we turn to two distributions defined over the half line $[0, \infty)$, the log-normal distribution $\frac{1}{x\sqrt{2\pi}\sigma} e^{-\frac{log(x-\mu)^2}{2\sigma^2}}$ whose mean is $e^{\mu+\frac{\sigma^2}{2}}$ and variance is is σ^2 and the Gamma distribution $G(x, a, b) = x^{a-1}\frac{exp(\frac{-x}{b})}{\Gamma(a)b^a}$ whose mean is ab and whose variance is ab^2. The natural logarithm of the Omega function for the Gamma distribution $G(x, a, b)$ is illustrated in Figure 4 for $a = 1$,b=4, $a = 2$, $b = 2$ and $a = 4$, $b = 1$. These distributions have a common mean of 4 and standard deviations of 16, 8 and 4 respectively. The derivatives of $log(\Omega)$ are shown in the bottom panel.

Although the Omega functions for lognormal and Gamma distributions with $a > b$ have similar qualitative features, as do the distributions themselves,they are quite different in detail. We illustrate this with a comparison of the two Omega functions for the case of the Gamma distribution $G(r, 2, 1)$ and a log-normal with the same mean and variance in Figure 5.

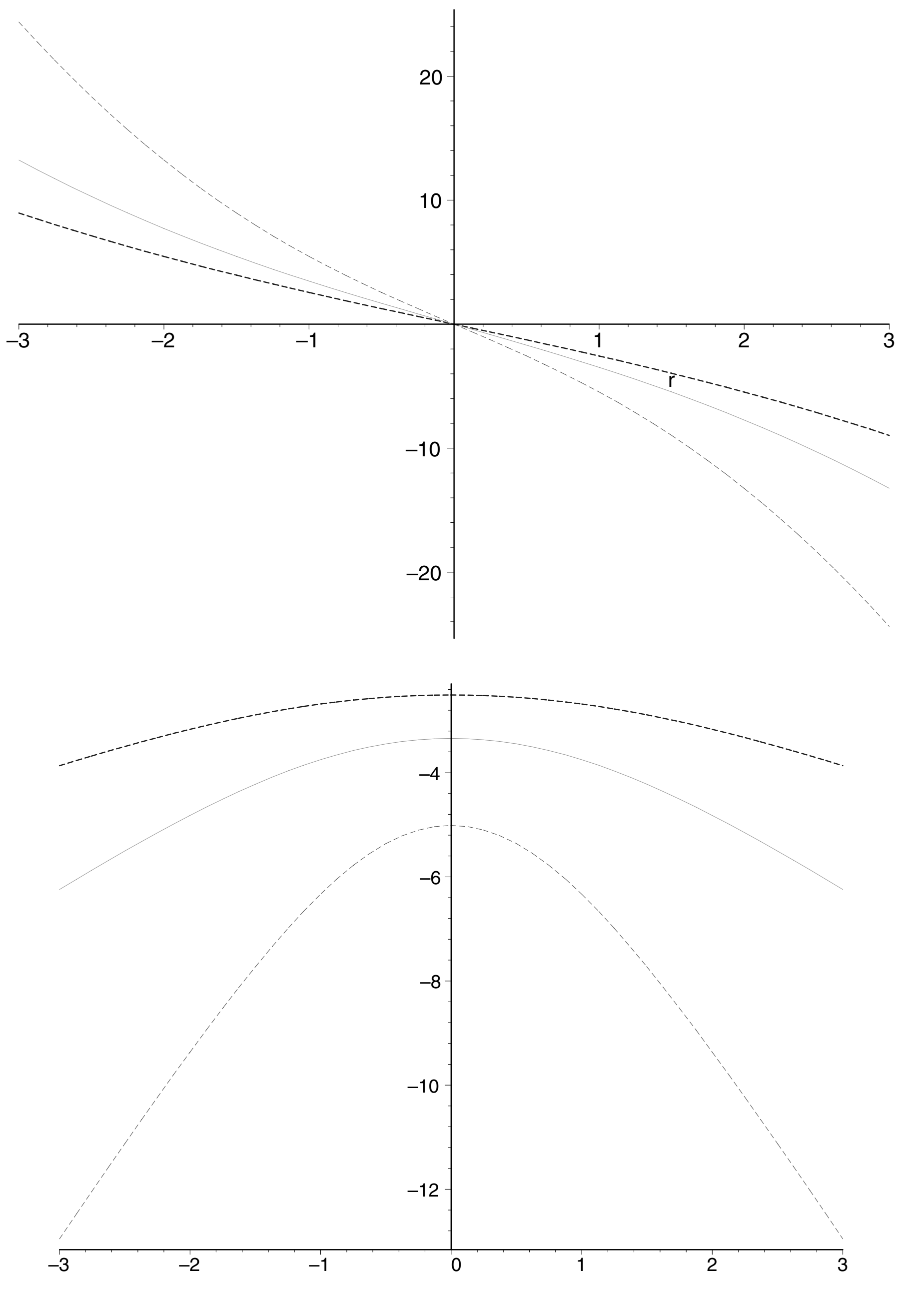


Figure 1: Top panel $Log\Omega(r)$ for normal distributions with common mean of $\mu = 1$ and $\sigma = 1.5$ (solid), $\sigma = 2$ (dashed) and $\sigma = 2.5$ (heavy dashed line). Bottom panel $\frac{1}{\Omega(r)} \frac{d\Omega}{dr}$ for the same values of σ.

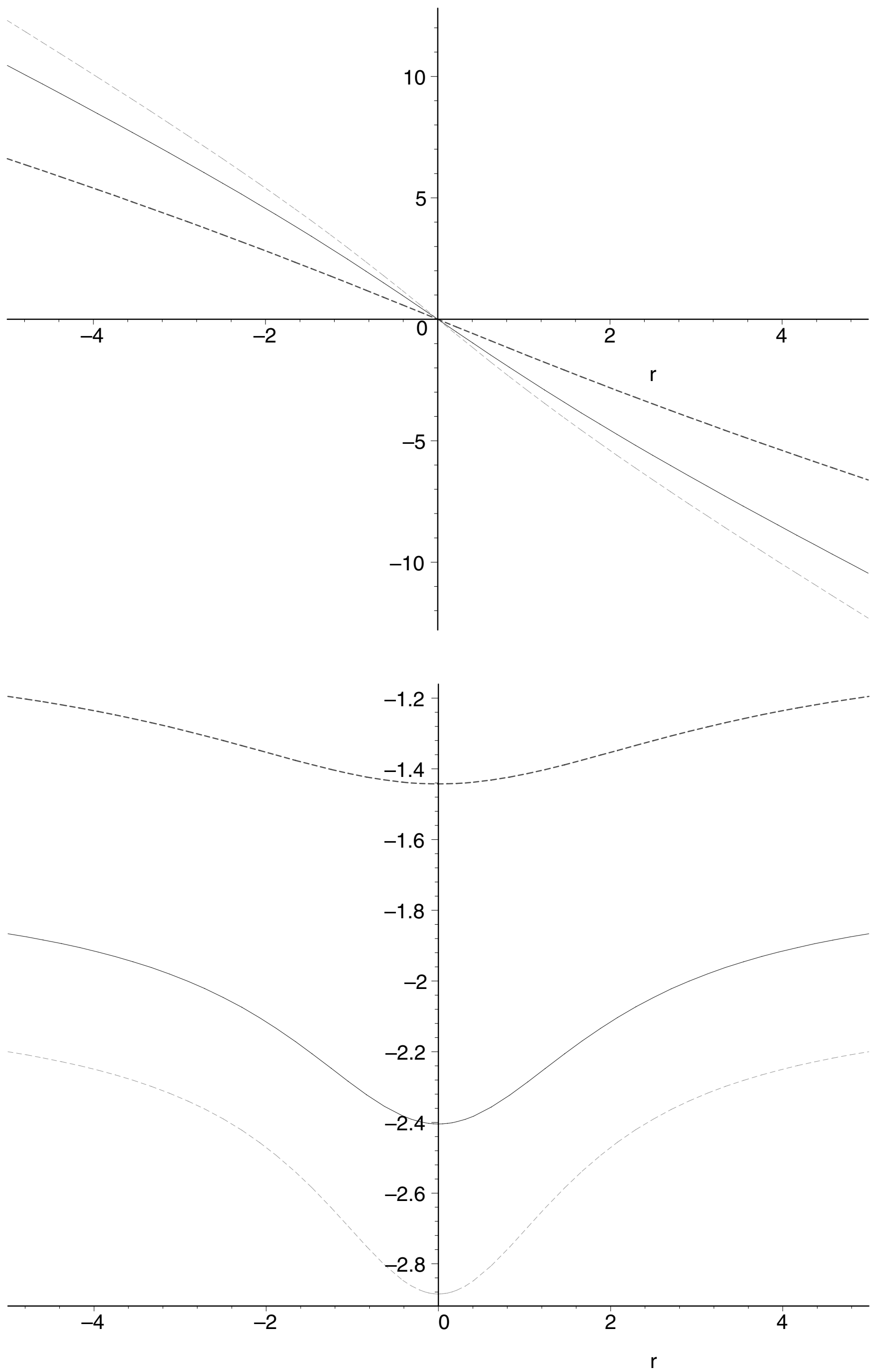


Figure 2: Top panel $Log\Omega(r)$ for logistic distributions with common mean of $a = 0$ for $b = 0.5$ (dashed), $b = 0.6$ (solid) and $b = 1$ (heavy dashed line). Bottom panel $\frac{1}{\Omega(r)}\frac{d\Omega}{dr}$ for the same values of b.

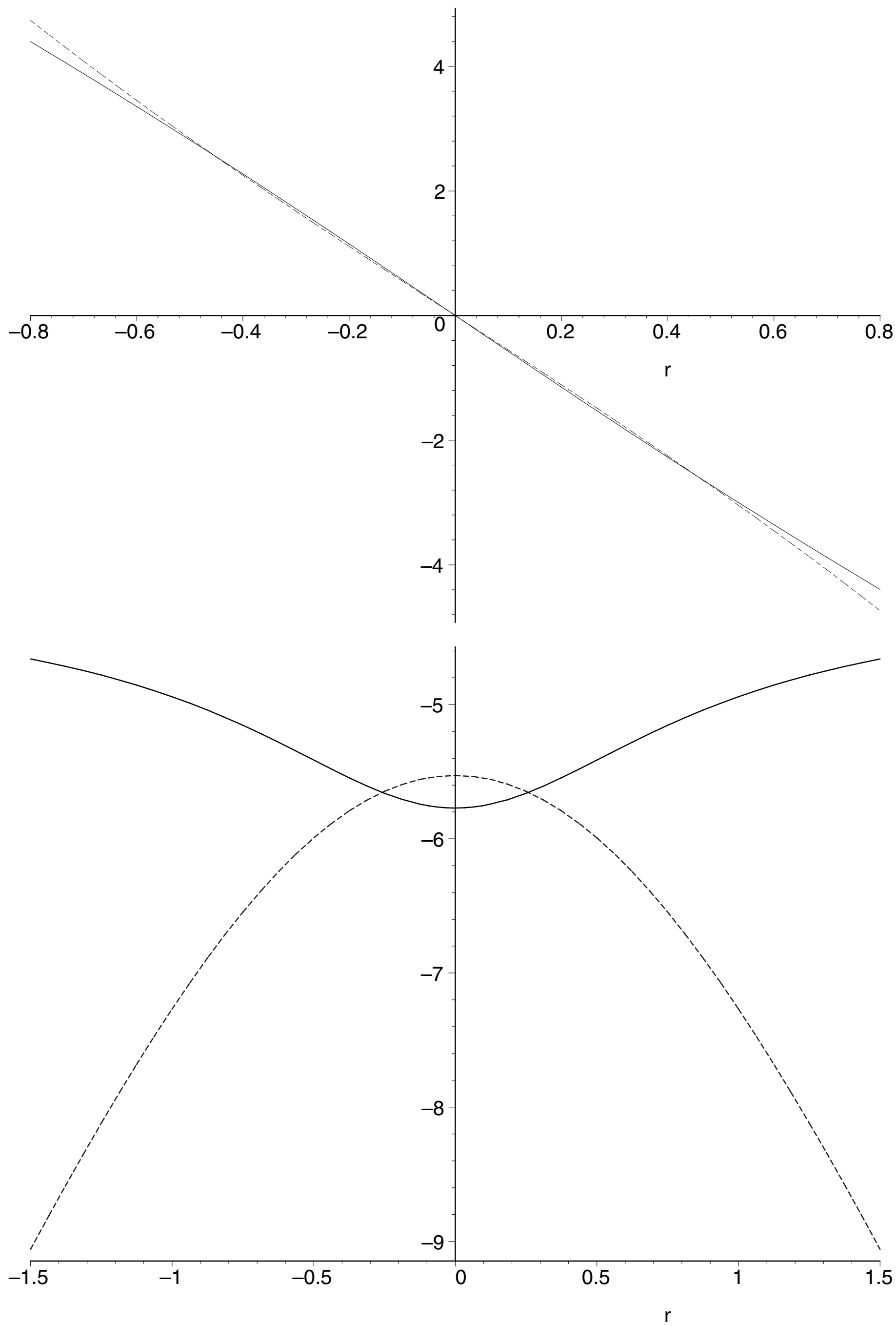


Figure 3: Top panel $Log\Omega(r)$ for a logistic distribution (solid) and a normal distribution (dashed line) with the same mean and variance. Bottom panel $\frac{1}{\Omega(r)}\frac{d\Omega}{dr}$ for the same distributions.

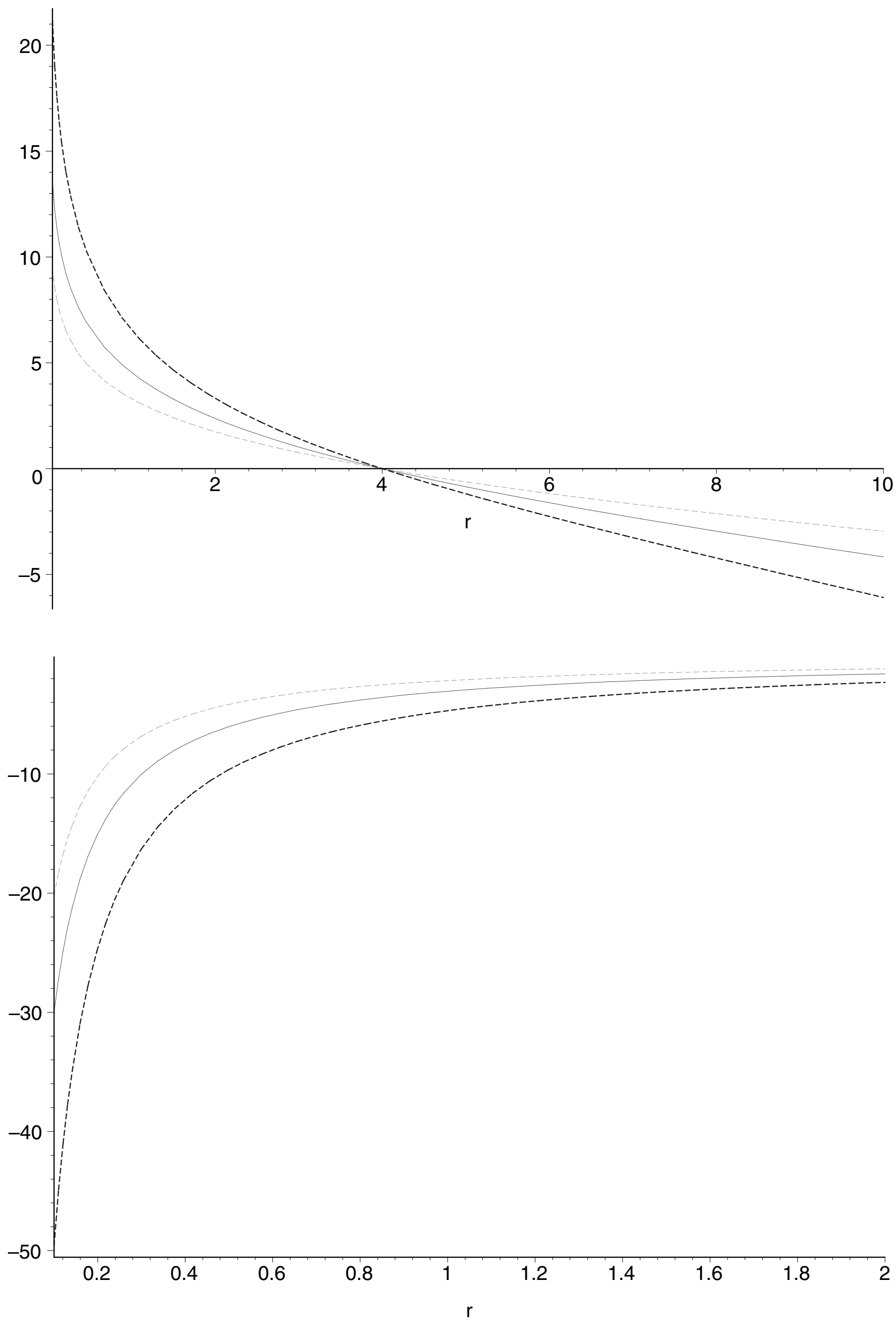


Figure 4: $Log\Omega(r)$for $Gamma(r, 1, 4)$ (dashed line),$Gamma(r, 2, 2)$ (solid line) and $Gamma(r, 4, 1)$(heavy dashed line). Bottom panel $\frac{1}{\Omega(r)}\frac{d\Omega}{dr}$for the same distributions.

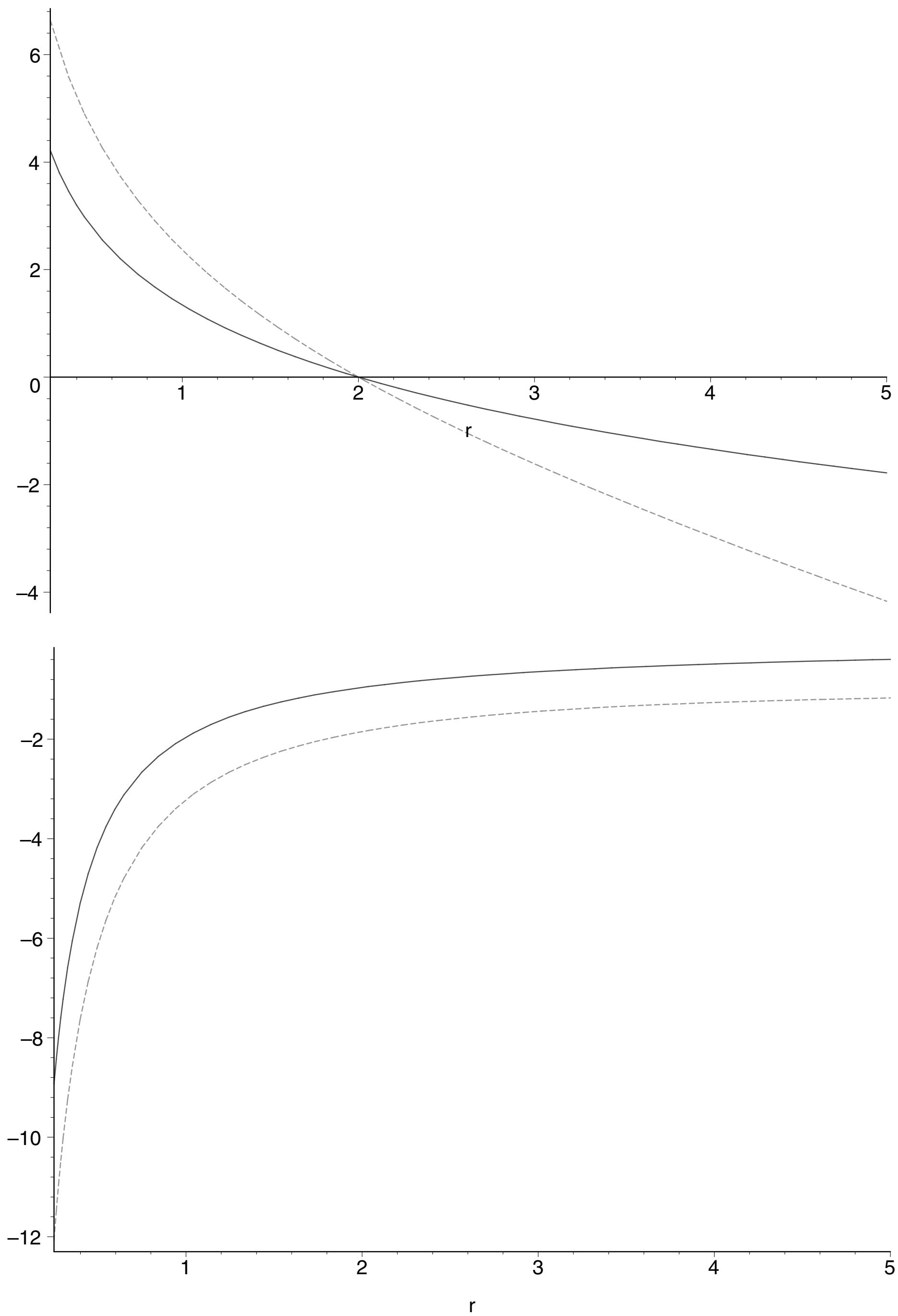


Figure 5: $Log\Omega(r)$ for $Gamma(r,2,1)$ (dashed), and a lognormal with mean and variance of 2 (solid line). Bottom panel $\frac{1}{\Omega(r)}\frac{d\Omega}{dr}$ for the same distributions.

4 An Example with Financial Returns Data

We illustrate the use of the Omega function of a distribution for the comparison of returns from two equity indices, the FTSE 100 index and the S&P500 index. The comparison is made on the basis of two years of daily index data from 3 January 1995 to 31 December 1996. The data sets consist of 505 returns. (Here the ith return is $r_i = \frac{Index(i+1)-Index(i)}{Index(i)}$, where $Index(i)$ is the closing level on day i.)

Comparison of the Omega functions for the returns distributions over this period provides an indication of the relative quality of investments in the two indices.

As the top panel of Figure 6 shows, the Omega function for the S&P500 index has heavier tails than the FTSE 100 index as well as a higher mean return. The observed difference in the two Omega functions is consistent with the standard statistics for the two data sets. The skewness of the S&P 500 returns set is -0.397 , compared with -0.157for the FTSE 100. The kurtosis of the S&P 500 returns set is 5.31 compared with 3.37 for the FTSE 100.

Over most of the observed range of returns, the S&P 500 Omega dominates the Omega function of the FTSE 100. This indicates that one might expect higher terminal values from an investment in the S&P 500 index than for an investment in the FTSE 100 index over this period. This was indeed the case, as the former produced a two year return of 61% while the latter produced a return of 34% over the same period. The subsequent year produced returns of 31.6% in the S&P 500 compared with 26.6% in the FTSE 100.

The standard models in finance assume that returns on financial instruments are normally distributed. The Jarque Bera test statistic for the FTSE 100 data is 4.93 while that for the S&P 500 is 135. As the sample contains 505 data points, we may reject the hypothesis that these returns are normally distributed at the 95% confidence level.

We may also compare the Omega functions for these data sets with those for normal distributions with the same mean and variance. As the bottom panel of Figure 6 and the top panel of Figure 7 show, the departure from the Omega function of a normal distribution is more pronounced for the S&P 500 returns than for those of the FTSE index. Moreover, both are substantially greater than the departure from normal of that of a data set consisting of 505 random draws

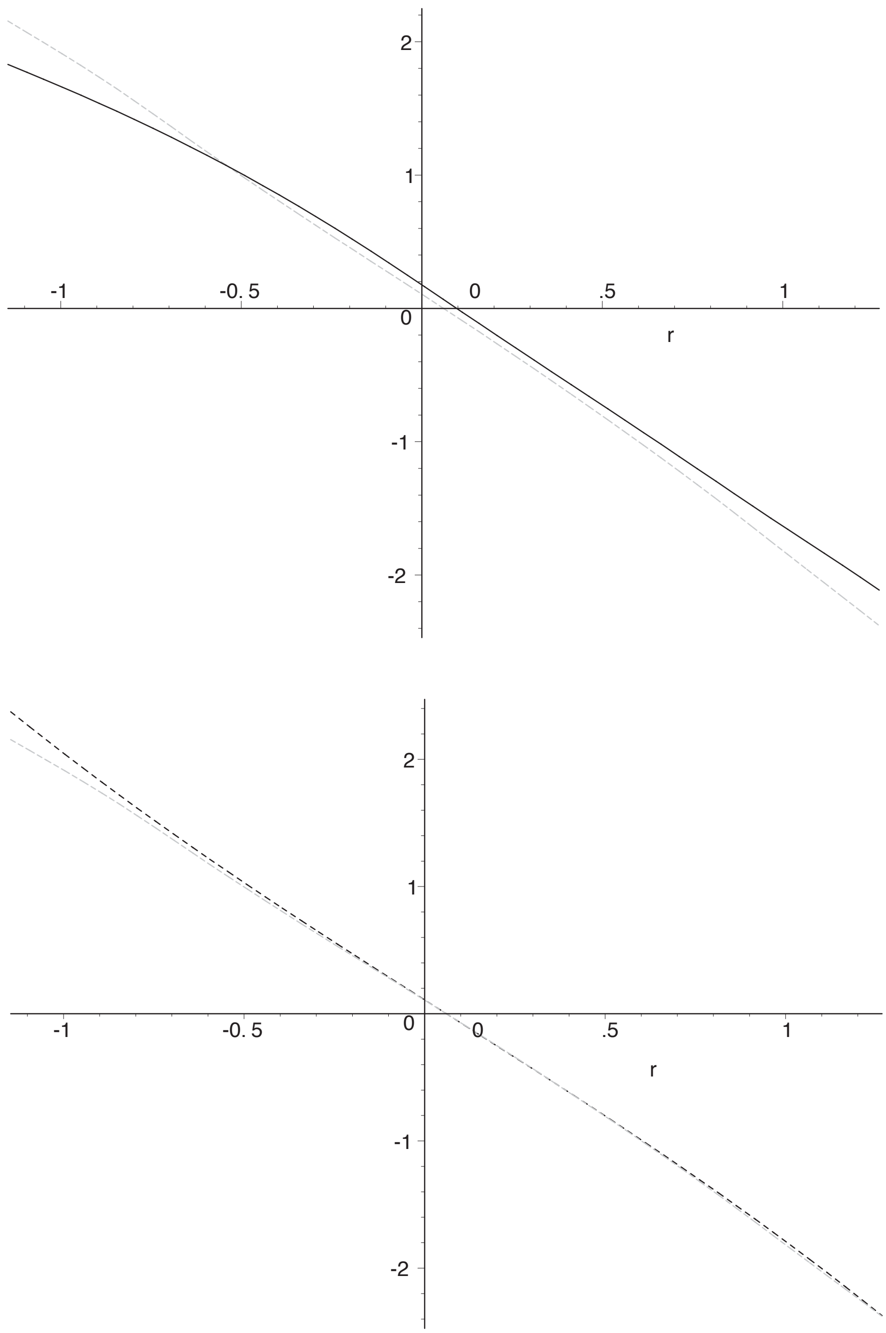
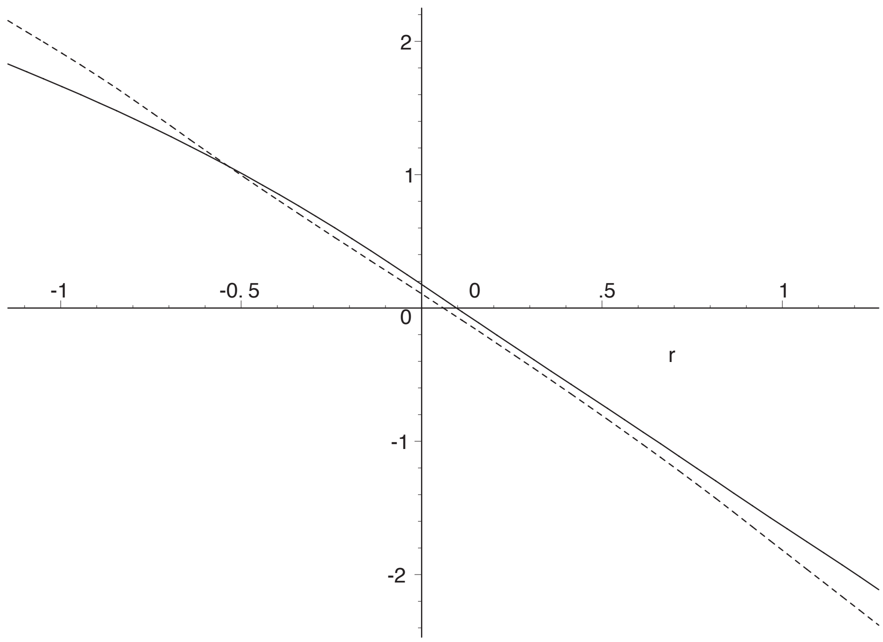


Figure 6: Top Panel $Log\Omega(r)$ for the FTSE 100 index (dashed), and the S&P500 index from 3 January 1995 to 31 December 1996. The range of returns is the FTSE mean return $+/-$ 2 FTSE standard deviations. Bottom Panel $Log\Omega(r)$ for the FTSE 100 index (dashed) and a normal distribution with the same mean and variance (heavy dotted line).



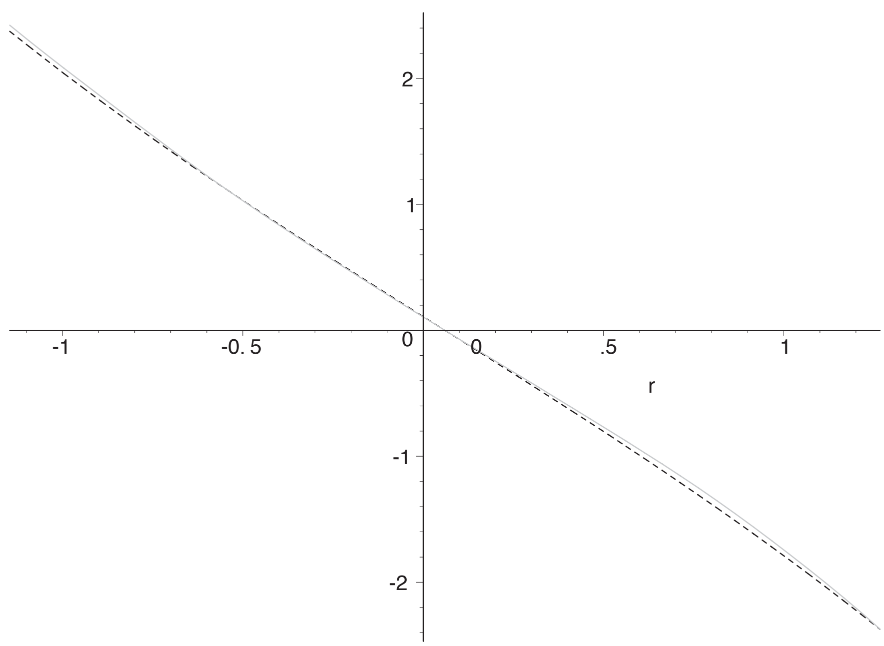


Figure 7: Top Panel $Log\Omega(r)$ for the S&P 500 index (solid), and a normal distribution with the same mean and variance (heavy dotted line). Bottom Panel $Log\Omega(r)$ for a sample of 505 data points from a normal distribution with the mean and variance of the FTSE 100 index (solid) and the true Omega for a normal with the same mean and variance.

from a normal distribution of the same mean and variance. This is illustrated in the bottom panel of Figure 7.

5 Further work

Of the many questions which are raised by this new way of looking at distributions, perhaps the most obvious is the question of what distributions arise from "natural" conditions on Omega functions.The inverse problem of constructing the distribution which produces a given Omega function is solved in [3] where a closed form expression for the distribution in terms of Omega and its first two derivatives is obtained. This, in turn, leads to some insights into the nature of the Ω-characterisation of natural distributions.

The question of the Ω-characterisation of the normal distribution is a natural one, to which we do not as yet have a satisfactory answer. We do, however, know the sampling distribution of the slope of the Omega curve for a normal distribution at the mean, as this is $-\frac{\sqrt{2\pi}}{\sigma}$, as we observed in section 3. Additional statistics for Omega curves are the subject of continuing investigation.

The Omega function extends naturally to multivariate distributions and this is discussed in a separate paper [4].

6 Acknowledgements

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References

[1] C. Keating and W.F. Shadwick *A Universal Performance Measure*, J. Performance Measurement vol 6 no 3, pp 59-84, 2002.

[2] A. Cascon, C. Keating and W.F. Shadwick, *An Introduction to Omega*, Finance Development Centre Tech. Report, 2002.

[3] A. Cascon, B.A. Shadwick and W.F. Shadwick *The Inverse Problem for Omega*, Finance Development Centre Tech. Report, 2002.

[4] A. Cascon and W.F. Shadwick *The Omega Function for a Multivariate Distribution*, Finance Development Centre Tech. Report, 2002.

FTSE 100 Returns									
-0.4599	1.5458	0.6387	-0.2072	0.1076	0.4346	0.6377	0.4279	0.1805	0.5751
-0.6325	-0.1551	0.1317	-0.7527	0.0707	-0.5456	-0.2058	-0.0479	0.1854	0.0721
1.0784	1.1297	0.9447	0.0872	-0.5144	-0.7783	0.35	0.1838	0.9251	-0.79
-0.3002	0.3457	0.1007	0.601	-0.6165	-0.4521	0.2352	-1.4228	-0.2375	-0.7412
0.1505	0.1499	-0.2722	0.1587	0.3459	0.0657	-0.6771	0.5935	0.5047	0.3835
-0.3594	-0.1051	0.2967	1.1295	0.5442	0.1476	-0.5704	0.7214	-0.2858	-0.769
-0.5313	0.542	-1.2748	0.7379	-0.2947	1.3078	0.3141	0.3595	0.4907	-0.5167
0.4978	-0.1142	0.2067	-0.1556	0.1222	-0.1725	0.0054	-0.199	0.4215	-0.1782
0.9317	-0.6826	0.1017	-0.4051	-0.6415	1.2013	0.7379	-0.2153	-0.0461	0.3723
-0.7248	0.4475	-0.2449	0.1166	0.4114	-0.008	0.5054	0.2105	0.3328	-0.8969
0.0164	1.0788	0.9162	0.1392	0.2162	0.1547	0.2716	-0.1356	-0.86	0.2666
-0.8609	-1.2058	-0.1276	-0.6326	0.4173	-0.5114	-0.0912	-0.0852	-0.4479	0.0921
-1.1094	0.1657	0.4516	0.04	0.1272	0.6212	0.8188	-0.6847	0.4344	0.5084
-1.3623	1.4317	-0.1213	-0.7477	-0.5139	-0.6386	0.0799	-0.1395	-0.7337	-0.1881
0.501	0.0659	0.0326	0.9086	0.3803	0.0134	0.2341	-0.3089	0.4357	-0.0204
0.4446	0.3354	0.7548	0.379	0.3138	-0.0027	-0.6158	-0.8435	0.3744	0.8176
0.8417	0.3124	-0.7168	0.2754	0.6624	0.0187	0.6036	0.4348	0.1492	0.0985
0.4955	-0.2087	-2.0773	0.7162	1.1032	0.6425	0.6292	-0.4519	0.4572	0.4038
-0.8702	-0.4151	0.1209	-0.3401	-0.0443	-0.1729	0.9181	0.8753	0.1355	-0.3846
-0.1435	0.5923	-0.9206	0.251	0.5431	0.7594	-0.515	0.3934	-0.2681	-0.2271
0.8591	-0.0312	0.3442	-0.1463	-0.6807	-0.9177	0.3968	0.0027	0.6913	1.6415
0.5767	-0.4456	0.6254	-0.3775	0.7852	0.024	0.9527	-0.3114	0.8976	0.8933
0.8238	-0.7638	0.2745	0.987	-0.8231	-0.571	-0.1141	1.252	0.2344	0.3404
0.0752	0.1451	0.7672	-0.1512	0.5968	-0.475	-0.5113	-0.4627	-0.1232	-0.4719
0.3494	0.7938	1.3645	-0.0224	0.6898	0.213	-0.4018	-0.0454	-0.4179	0.0247
-0.0065	0.2938	-0.1944	-0.8781	-0.0055	0.2772	0.0445	0.2887	0.4702	0.1926
0.8625	0.1745	2.2017	0.2292	0.1836	0.5635	0.3535	0.3598	-0.2566	-0.4805
0.3517	0.3515	-0.2281	0.5676	0.2407	-0.0694	-0.6157	-0.4461	-1.1201	0.5695
-0.9261	-0.2666	0.2605	-0.1011	0.4394	0.9292	0.2284	-0.5521	-0.2347	-0.4013
-0.3181	-0.028	-0.3868	-1.2114	-0.2907	-0.2355	-0.3117	-0.8047	0.6444	0.1483
0.1172	0.115	-0.0985	-0.2219	-0.1499	-0.7054	-0.7777	-1.7846	-0.0635	-2.1771
-0.774	0.8632	-0.5222	0.4648	-0.0382	-0.7932	-0.6567	0.713	0.3356	1.2263
-0.2261	0.4433	0.3908	-1.087	-0.6361	0.2961	-0.7623	0.9622	0.185	0.6008
-0.8409	0.0521	-0.6361	-0.1722	-0.261	0.3865	-0.4217	0.463	0.9738	-1.3182
0.159	-0.386	-0.4502	0.8393	0.6088	0.008	0.5664	-0.787	0.5736	0.2059
-0.129	0.2922	-0.1439	0.3421	0.0767	-0.9652	0.7001	0.7362	-0.3761	-0.4586
0.9869	0.8862	0.3735	0.1136	0.2052	0.3159	-0.4049	-1.0678	0.625	0.5387
-0.3804	0.845	0.542	0.5647	0.2512	0.6001	0.5482	0.4334	0.1615	-0.3556
-0.4082	-0.2291	0.0379	0.0085	-0.7898	-0.2836	0.4389	-0.3094	0.1017	0.9699
-0.5289	0.0121	0.6234	-0.505	-1.2766	0.6734	-0.5985	0.1497	-0.6517	0.8238
1.06	-0.299	0.1158	-0.4594	-0.5339	0.4237	0.9591	-0.28	-0.3642	0.6492
-0.0986	-0.103	0.3065	-1.4301	1.0288	0.2255	-0.3008	0.9459	0.8361	0.2354
-0.4312	-0.3518	-0.1614	0.4104	0.5424	-0.4819	0.2964	0.8425	0.2632	0.1297
-0.7669	-0.7548	-0.3869	1.4247	0.6881	-0.0186	-0.665	0.9694	0.2996	-0.0367
-0.8295	0.7114	1.4493	1.2543	0.4948	-1.2745	-0.4569	0.4694	-0.6517	0.6038
0.5072	0.2314	-0.6943	-0.2999	0.0082	-0.9649	0.1361	0.0026	0.4398	0.068
-0.1738	1.0784	0.1956	0.1377	0.3427	-0.9525	0.2159	0.5992	0.2721	
1.145	0.027	0.0316	0.8646	-0.0379	0.022	0.4122	0.0079	0.4934	
-0.3078	-0.5138	-0.422	-0.4008	0.7511	1.14	-0.7681	-0.0184	-0.3904	
1.2883	-0.0362	-0.0144	-0.7601	-0.0404	-1.0049	0.0294	-0.1942	-0.7098	
-0.118	0.287	0.1845	-0.5603	-0.2585	0.6804	-0.2295	0.5285	-0.7199	

Figure 8: Returns for the FTSE 100 index 3 January 1995 to 31 December 1996.

S&P 500 Returns									
0.3485	0.0222	1.8755	-0.4198	0.2564	0.2838	-0.2624	-0.2152	0.2802	-0.1951
-0.0803	0.1251	0.0169	0.8341	-0.6973	-1.4568	-0.1215	0.7309	0.0347	-0.5222
0.0739	-0.2157	-0.1837	-0.209	-0.9872	-1.8	0.2203	0.8743	0.4424	0.6081
0.0326	0.1212	0.5803	0.2506	0.5167	0.7034	-0.0891	-0.7974	0.206	-0.0855
0.1844	0.0565	-0.0093	-0.1661	0.6124	-0.146	0.4507	0.0416	-0.1335	0.6235
-0.0022	1.0122	-0.4519	0.0304	-0.3	-0.3307	-0.6218	-0.1708	-0.0931	-0.2127
-0.0065	0.4451	-0.1463	-0.1967	0.4678	1.4371	0.0046	-0.177	0.8435	0.4206
0.938	0.1391	-0.8284	0.2526	0.9414	-0.3402	-0.5301	-0.2876	-0.5442	1.0485
0.7318	-0.1548	0.5569	-0.4254	0.1441	0.3084	1.275	-0.1674	-0.4722	1.4633
0.1427	-0.1789	0.9739	0.0574	-0.3573	0.5902	0.234	-0.3099	0.3796	0.4223
-0.0723	-0.3007	0.0784	0.4736	-0.3637	0.2566	0.0946	-0.1036	-0.2386	0.4356
-0.5876	0.2277	0.1212	-0.1875	0.9193	-0.0994	-0.003	-0.4661	-1.1146	0.1437
-0.4647	0.6755	0.5045	0.1699	0.262	1.1701	-1.7717	-0.0151	-0.8229	-0.3156
0.2238	0.0653	0.9985	0.1643	-0.091	-0.4726	-0.3182	0.0211	0.4187	0.2152
0.0086	0.1009	-0.044	0.1711	-0.0709	0.7439	-1.3532	0.7159	0.1359	0.6497
0.3392	0.0672	-0.1835	0.3488	-0.5082	0.4183	-0.3662	0.3014	-0.9411	0.2365
0.1883	0.1165	1.3034	0.9453	0.7925	0.95	0.8761	-0.0553	0.9608	-0.0813
0.442	-0.2919	-0.2468	0.1757	0.5691	0.9315	0.9078	-0.6118	1.2323	0.6974
-0.3997	0.3244	-1.0151	0.021	0.457	0.3836	0.3907	0.6261	0.0075	0.2412
0.4077	0.4062	-0.3124	0.4191	-0.5366	-0.4088	-0.5256	0.3111	0.5227	-0.1613
-0.0043	-0.6088	0.424	0.2148	0.568	0.8776	0.3117	0.7828	0.58	0.8051
0.5081	-0.1502	-0.1579	0.453	-0.3065	0.7639	0.2268	-0.3359	1.3991	1.1085
1.2395	-0.089	0.1618	0.392	0.2624	0.557	0.4372	-0.1796	0.5055	-0.1413
0.5202	0.0733	0.4296	0.8363	0.225	0.9447	0.5695	-2.2249	-0.1521	-0.127
-0.0686	0.6333	0.0311	-0.0446	0.8531	0.0457	-0.2164	-0.7453	-0.2152	0.2675
0.079	0.8653	1.2298	-0.0994	0.1962	0.774	0.4153	0.3387	0.2245	-0.0608
-0.2078	-0.1443	0.4296	0.2454	-0.3736	-0.1421	0.0904	0.2001	0.5886	-1.0944
0.2645	0.0996	0.1474	0.4399	0.266	-0.7464	0.1071	-1.5837	-0.0786	-0.425
0.0395	0.1736	-0.4325	-0.6425	1.1038	-0.6498	0.0015	0.0805	-0.1267	-0.0966
0.1869	0.2259	1.1013	-0.2178	0.6518	-0.5128	0.0627	-2.5364	0.0321	-0.6421
0.4124	-0.0874	0.0196	0.0138	0.4047	-1.1312	-1.711	-0.2271	0.0044	1.3737
0.1403	0.1167	-0.1979	-0.0688	-0.6466	1.1629	-0.272	0.9071	0.0481	-0.2961
-0.6698	1.0916	0.5055	-0.0636	0.2126	1.6602	-0.1278	1.4967	0.1632	-0.911
0.1556	0.0115	-0.757	0.8313	0.3304	0.0334	-0.3979	-0.7505	0.2575	-1.5431
0.4868	-0.0807	-1.3394	-0.2492	-0.1194	-1.3079	1.0215	-0.7765	0.7154	-0.0905
0.3793	0.7383	0.4646	-0.4603	0.4703	-0.495	0.1024	-1.0887	-0.1772	-1.0513
0.2773	-0.0763	0.0145	0.1066	-0.7673	-0.3847	1.0303	-0.0351	1.2529	0.7018
-0.9114	0.1528	0.5437	-0.1494	-0.094	-0.67	1.4446	0.7213	0.2737	0.7575
0.74	0.0019	0.803	0.1995	-1.5462	0.6152	0.6183	0.7494	-0.3895	1.9438
-0.357	0.225	0.0909	-0.024	0.8438	0.9994	-0.027	-0.7847	-0.5566	0.417
-0.1071	0.4167	0.6428	-0.7073	-0.9789	0.7652	-0.0857	0.6895	-0.3057	-0.2604
0.0598	0.0853	-0.4052	-0.147	0.7509	-0.5779	0.6107	0.7383	0.871	0.5503
0.0433	-0.212	-0.1545	0.3359	0.2408	0.2531	0.6339	1.5736	0.411	0.6378
-0.7228	-1.4211	-0.4306	0.6282	0.3824	-3.0827	-0.0579	1.9184	-0.1379	0.1283
0.2116	-0.0751	-0.1501	0.2401	0.0374	1.0292	0.8413	-0.3411	0.2619	-0.3885
0.0041	0.859	-0.0089	-0.2515	-0.0667	-0.4578	-0.3567	0.3256	0.3663	-1.7391
1.3267	0.9434	0.034	0.6432	0.2947	0.2292	0.3713	0.2687	0.5417	
0.098	0.0038	0.195	0.1125	0.7793	0.3633	-0.9256	-0.2364	-0.1365	
0.5795	-0.0038	0.0643	0.5464	0.095	0.0874	-0.6397	-0.074	-0.4621	
-0.2049	-0.9346	-0.1213	-0.5401	-0.5826	1.7492	0.5644	0.5543	0.0991	
0.7177	-0.0134	-0.4038	-0.4085	-0.1603	-0.1471	-0.3841	-0.8366	-0.7041	

Figure 9: Returns for the S&P 500 index 3 January 1995 to 31 December 1996.