# THE OMEGA MEASURE:
# HEDGE FUND PORTFOLIO OPTIMIZATION

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# 1 Introduction

Traditionally, asset allocation is performed in the mean-variance framework and performance evaluation is formalized in the same terms with the Sharpe ratio. Mean-variance analysis requires assumptions on the investor's utility function, namely a quadratic utility function, or on the normality of the returns distribution. It is well known however that a quadratic utility function is inconsistent with rational human behavior. Moreover, hedge fund returns are usually far from normally distributed. Thus, mean and variance do not appropriately capture the risk and reward properties of hedge fund returns and alternative methods capturing their asymmetry and fat tails have been introduced. However, despite the improvement they can add to the analysis, these methods still reduce the dimensionality to a few characteristics and do not take into account moments of higher order than skewness and kurtosis.

It can be shown that investors care about all the moments of the distribution, which is of great importance when returns are not normally distributed. Omega is a new measure proposed by Keating and Shadwick (2002a, 2002b) that reflects all the statistical properties of the returns distribution, i.e. all the moments of the distribution are embodied in the measure. It requires no assumptions on the returns distribution or on the utility function of the investor.

There is evidence that a diversified investment portfolio can benefit from the inclusion of hedge funds. While quantitative analysis provides appropriate tools for the identification of attractive investment opportunities in traditional asset classes, this is no longer the case for the emerging alternative asset classes where the methods are more exploratory. The objective of this paper is to analyze the empirical properties of omega as a measure of attractiveness and to apply it to hedge fund portfolio optimization. In order to evaluate the benefits of the measure, we contrast the results derived in the omega framework with those of other conventional settings. We find that higher moments than skewness and kurtosis can be of great significance for hedge fund index ranking. Regarding the portfolio optimization problem, we show that omega forms a convex space in a risk-reward representation. We find that frameworks with no assumption on normality tend to exhibit weights of comparable magnitude, and these weights tend to be quite different from those obtained with measures assuming normality. Our results suggest nevertheless that omega provides a more efficient frontier than any other setting and shows improved risk diversification and performance enhancement capabilities.

The document is organized as follows. Chapter 2 introduces the concept of investor preferences and shows the theoretical justification for the inclusion of higher moments in portfolio analysis. Chapter 3 is dedicated to the statistical properties of hedge fund indices and presents the data used for the empirical applications. In Chapter 4, we present the main frameworks commonly used for the investment decision problem and discuss their weaknesses. In Chapter 5, we introduce the omega function and discuss its properties and applications; a large part of the section is devoted to the comparison of hedge fund performance with different measures. In Chapter 6, we derive efficient frontiers of optimal portfolios in the different frameworks and try to compare their efficiency. Finally, Chapter 7 presents our conclusions.

## 2 Investors' Preferences in Portfolio Theory

A utility function reflects an investor's preference ordering and also reveals its attitude to risk. For a wealth level $W$, a rational risk-averse investor is assumed to have a strictly concave utility function and a positive marginal utility. Thus the first two derivatives of the function are the following:

$$\begin{aligned} U'(W) &> 0 \quad \forall W \\ U''(W) &< 0 \quad \forall W \end{aligned}$$

The fact that investor's marginal utility is positive implies that he always prefers more wealth to less. In the investment decision problem, the investor's objective is to maximize its utility function. Scott and Horvath (1980) show that investors care about moments of higher order than the variance. Under relatively weak assumptions[1], they demonstrate that investors like (positive) skewness and dislike kurtosis. More generally, they like odd moments and dislike even ones. To justify the inclusion of higher moments in the portfolio analysis, we can use the Taylor's series expansion of an investor's utility of wealth around its expected wealth,

$$U(W) = U[E(W)] + U'[E(W)][W - E(W)] + \sum_{j=2}^{\infty} \frac{1}{j!} U^{(j)}[E(W)][W - E(W)]^j$$

Computing the expected utility of wealth gives,

$$E[U(W)] = U[E(W)] + U'[E(W)]\underbrace{[E(W) - E(W)]}_{=0} + \sum_{j=2}^{\infty} \frac{1}{j!} U^{(j)}[E(W)]E[W - E(W)]^j$$

or equivalently,

$$E[U(W)] = U[E(W)] + \sum_{j=2}^{\infty} \frac{1}{j!} U^{(j)}[E(W)]E[W - E(W)]^j \cdot$$

Rewriting the first four terms, we obtain,

$$E[U(W)] = U[E(W)] + \frac{1}{2} U''[E(W)]\sigma^2(W) + \frac{1}{3!} U'''[E(W)]s^3(W) + \frac{1}{4!} U^{(4)}[E(W)]k^4(W) + \sum_{j=5}^{\infty} \frac{1}{j!} U^{(j)}[E(W)]E[W - E(W)]^j$$

where $E(W)$ is the mean of the wealth distribution and $\sigma^2(W)$ the variance. $s^3(W)$ is the third central moment and is directly linked to the skewness. The fourth central moment, $k^4(W)$, relates to the kurtosis. Analogously, the following terms of the equation represent the moments of higher order.

Traditionally, the problem is solved in the mean-variance framework developed by Markowitz (1952). However, this framework assumes that either the returns are normally distributed, in which case mean and variance are sufficient to fully characterize the distribution, or investor's utility function is quadratic. The general form of a quadratic utility function can be written as follows:

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[1] The assumptions are that investors exhibit positive marginal utility and consistent risk aversion at all wealth level, as well as strict consistency of moment preference

$$U(W) = a + bW + cW^2 \quad \text{with } a > 0 \text{ and } \quad c < 0.$$

If the investor's utility function is quadratic, its expected utility is only function of the first two moments of the distribution. Thus mean and variance are sufficient to solve the maximization problem even though returns are not normally distributed. This can be seen in the Taylor's series expansion where the third and the following terms are equal to zero since the second derivative of the quadratic utility function is a constant. However, this type of function does not make sense for a rational investor. Indeed, the function allows for negative marginal utility (satiation) and implies increasing absolute risk aversion (IARA).

In a recent paper, Siegmann and Lucas (2002) develop a model linking individuals' loss averse utility function with the particular pay-off structure of hedge funds. The model provides a framework where the investor faces a trade-off between maximizing its wealth and minimizing the expected shortfall below a specific target level. Their results suggest that financial optimization with this model closely matches common patterns observed in hedge fund empirical returns.

# 3 Statistical Properties of Hedge Fund Returns

## 3.1 Basic Features

It is well known that hedge fund returns are in general not normally distributed. Brooks and Kat (2001) show that monthly returns of hedge fund indices typically exhibit negative skewness and high kurtosis properties. In other words, large negative returns are more likely to occur than what would be predicted under conditions of normality. As documented by Moix and Schmidhuber (2001), hedge fund returns are more accurately described by a hyperbolic distribution. Figure 3.1 contrasts the histogram of typical hedge fund returns with a normal density.

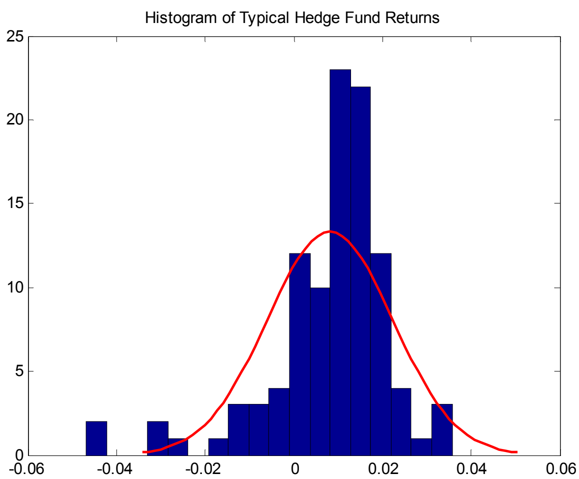


Figure 3.1: Monthly returns of the CSFB-Tremont Convertible Arbitrage Index for the period between January 1994 and July 2002. The returns exhibit a mean of 0.82%, a standard deviation of 1.41%, skewness of -1.6 and kurtosis of 6.9.

## 3.2 Historical Data

Every hedge fund manager tends to follow its own investment policy, which means that hedge fund styles are very heterogeneous. However, hedge fund data suppliers classify hedge funds into different broad categories and several subcategories (see Amenc and Martellini (2002b) for a review of the world of hedge fund indices). It is well known that there exist great disparities among competing hedge fund data suppliers even for indices of the same type. These discrepancies may arise for several reasons, mostly because different hedge funds are included in the indices, because the weighting schemes are not the same or simply because of different style classification.

For our empirical analysis, we use sets of data supplied by Credit Suisse First Boston-Tremont (CSFB-Tremont) and Hedge Fund Research (HFR). CSFB-Tremont indices are based on the TASS database and reflect the monthly net of fees returns of hedge funds on an asset weighted basis. Funds with relative big size have therefore a greater influence on the indices than smaller funds. CSFB-Tremont computes 9 strategy indices and 1 composite index. To limit the survivorship bias, hedge funds are not excluded until they liquidate or fail to meet the financial reporting requirements. HFR indices reflect the monthly net of fees returns of equally weighted hedge funds. The HFR indices are broken down into 33 different categories by strategy, including composite indices and a Fund of Funds index. By including funds that ceased to exist, HFR indices tend to eliminate the survivorship bias problem.

Our CSFB-Tremont data set includes the 9 strategy indices as well as the composite Hedge Fund Index. For HFR, we select 14 non-overlapping HFR strategy indices as well as the Fixed Income composite index and the Fund of Funds index. To replicate portfolios with other financial asset classes such as bonds and equity, we also select the Salomon Smith & Barney World Government Bond Index 10-15 years maturity (WGBI) and the Morgan Stanley Capital Index world (MSCI). The data consists of monthly returns between January 1994 and July 2002, i.e. 103 data points for each index. Thus, the data includes market turmoil such as the 1997 Asian crisis, the 1998 Russian crisis and the subsequent LTCM collapse, as well as the 1999 Brazilian crisis. The data also incorporates the end of the IT bubble.

Table 3.1 shows the main statistical properties of the data. A more complete set of statistics with normalized central moments up to eight are provided in appendix 1. WGBI and MSCI are normally distributed according to the Jarque-Bera test at 95%. Not surprisingly, most of the hedge fund indices exhibit negative skewness and excess kurtosis. CSFB-Tremont indices show mean returns covering a range from 0.13% (Dedicated Short Bias) to 1.18% (Global Macro). The lowest standard deviation is displayed by Equity Market Neutral (0.93%) and the highest by Emerging Markets (5.48%). All the indices exhibit excess kurtosis except Equity Market Neutral. Among the leptokurtic indices, four have negative skewness. On the whole, normality is rejected for 6 indices with the Jarque-Bera test at 95%. The indices which most depart from normality are Event Driven and Fixed Income Arbitrage; they have the highest negative skewness and excess kurtosis properties. HFR indices mean returns cover a range from 0.48% (Fixed Income: Arbitrage) to 1.26% (Equity Hedge). The lowest standard deviation is displayed by Relative Value (0.74%) and the highest by Short Selling (7.08%). Most indices exhibit negative skewness and/or excess kurtosis and normality is rejected with the Jarque-Bera test at 95% for 11 of them, among which Distressed Securities, Event Driven, Fixed Income: Arbitrage, Merger Arbitrage and Relative value exhibit the highest negative skewness and kurtosis properties.

| Historical Data Statistical Properties | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Mean | Minimum | Maximum | Standard Dev. | Skewness | Kurtosis | JB Statistic |
| **CSFB-Tremont Indices** | | | | | | | |
| Hedge Fund Index | 0.89% | -7.55% | 8.53% | 2.63% | 0.1 | 4.1 | 4.3 |
| Convertible Arbitrage | 0.82% | -4.68% | 3.57% | 1.41% | -1.6 | 6.9 | **103.2** |
| Dedicated Short Bias | 0.13% | -8.69% | 22.71% | 5.31% | 0.9 | 5.0 | **28.7** |
| Emerging Markets | 0.55% | -23.03% | 16.42% | 5.48% | -0.5 | 5.8 | **34.4** |
| Equity Market Neutral | 0.89% | -1.15% | 3.26% | 0.93% | 0.1 | 2.9 | 0.2 |
| Event Driven | 0.86% | -11.78% | 3.68% | 1.84% | -3.3 | 22.9 | **1797.5** |
| Fixed Income Arbitrage | 0.59% | -6.96% | 2.02% | 1.16% | -3.5 | 20.6 | **1472.9** |
| Global Macro | 1.18% | -11.55% | 10.60% | 3.76% | 0.0 | 4.3 | 5.9 |
| Long-Short Equity | 1.00% | -11.44% | 13.01% | 3.40% | 0.2 | 5.5 | **24.8** |
| Managed Futures | 0.54% | -9.35% | 9.95% | 3.43% | 0.1 | 4.0 | 3.4 |
| **HFR Indices** | | | | | | | |
| Convertible Arbitrage | 0.88% | -3.19% | 3.33% | 1.04% | -1.3 | 6.0 | **61.6** |
| Distressed Securities | 0.82% | -8.50% | 5.06% | 1.66% | -1.8 | 11.8 | **363.4** |
| Emerging Markets (Total) | 0.58% | -21.02% | 14.80% | 4.68% | -0.7 | 6.6 | **60.0** |
| Equity Hedge | 1.26% | -7.65% | 10.88% | 2.89% | 0.3 | 4.3 | **7.0** |
| Equity Market Neutral | 0.78% | -1.67% | 3.59% | 0.98% | 0.0 | 3.2 | 0.1 |
| Equity Non-Hedge | 1.08% | -13.34% | 10.74% | 4.40% | -0.4 | 3.3 | 3.6 |
| Event-Driven | 1.03% | -8.90% | 5.13% | 2.01% | -1.3 | 8.0 | **131.7** |
| Fixed Income (Total) | 0.70% | -3.27% | 3.28% | 0.97% | -1.2 | 7.0 | **88.3** |
| Fixed Income: Arbitrage | 0.48% | -6.45% | 3.04% | 1.29% | -2.8 | 15.7 | **793.2** |
| Fixed Income: Convertible Bonds | 0.70% | -13.06% | 14.42% | 4.11% | -0.5 | 5.3 | **24.1** |
| Fund of Funds | 0.63% | -7.47% | 6.85% | 1.89% | -0.2 | 6.2 | **42.1** |
| Macro | 0.83% | -6.40% | 6.82% | 2.27% | 0.0 | 3.6 | 1.2 |
| Merger Arbitrage | 0.91% | -5.69% | 2.47% | 1.13% | -2.5 | 13.5 | **557.7** |
| Relative Value | 0.75% | -2.96% | 2.01% | 0.74% | -1.9 | 9.5 | **230.1** |
| Short Selling | 0.59% | -21.21% | 22.84% | 7.08% | 0.2 | 3.9 | 3.3 |
| Statistical Arbitrage | 0.70% | -2.00% | 3.60% | 1.14% | -0.2 | 3.0 | 0.7 |
| **Traditional Indices** | | | | | | | |
| SSB WGBI | 0.57% | -5.08% | 7.71% | 2.21% | 0.2 | 3.7 | 2.1 |
| MSCI | 0.40% | -13.45% | 8.91% | 4.13% | -0.6 | 3.3 | 5.5 |

Table 3.1: Historical Data Statistical Properties. The figures in bold in the Jarque-Bera statistics column indicate that normality is rejected at a 95% confidence interval.

# 4 Conventional Frameworks for the Investment Decision Problem

As hedge fund returns are usually far from normally distributed, mean and variance are not appropriate to capture the risk and reward properties of the returns distribution. Alternative methods capturing the asymmetry and fat tail properties of hedge fund returns have been introduced. In this chapter, we present commonly used frameworks for the investment decision problem and discuss their weaknesses. Appendix 2 shows the mathematical formulation of the risk and reward measures used in the different frameworks.

## 4.1 Mean -Variance Framework

The Mean-Variance analysis developed by Markowitz (1952) is the traditional approach used to evaluate investments in portfolios. The framework formalizes the idea that a risk averse investor faces a risk/reward trade-off for its investment decision problem. In this setting, the risk is defined as the variance (or the standard deviation) and the reward is represented by the expected return. The investor's utility function is therefore constrained to be only a function of the mean and variance of the returns distribution.

Asset allocation is performed via solving an optimization problem. For arbitrary levels of expected return, the weights of several assets are combined so that they form an optimal portfolio with minimum variance. Figure 4.1 represents the minimum variance frontier in the traditional risk/reward dimensions (standard deviation and expected return) for portfolios of 10 hedge fund indices (CSFB-Tremont indices) and 2 traditional asset indices (WGBI and MSCI). The optimization is performed with monthly returns and no short sales are allowed.

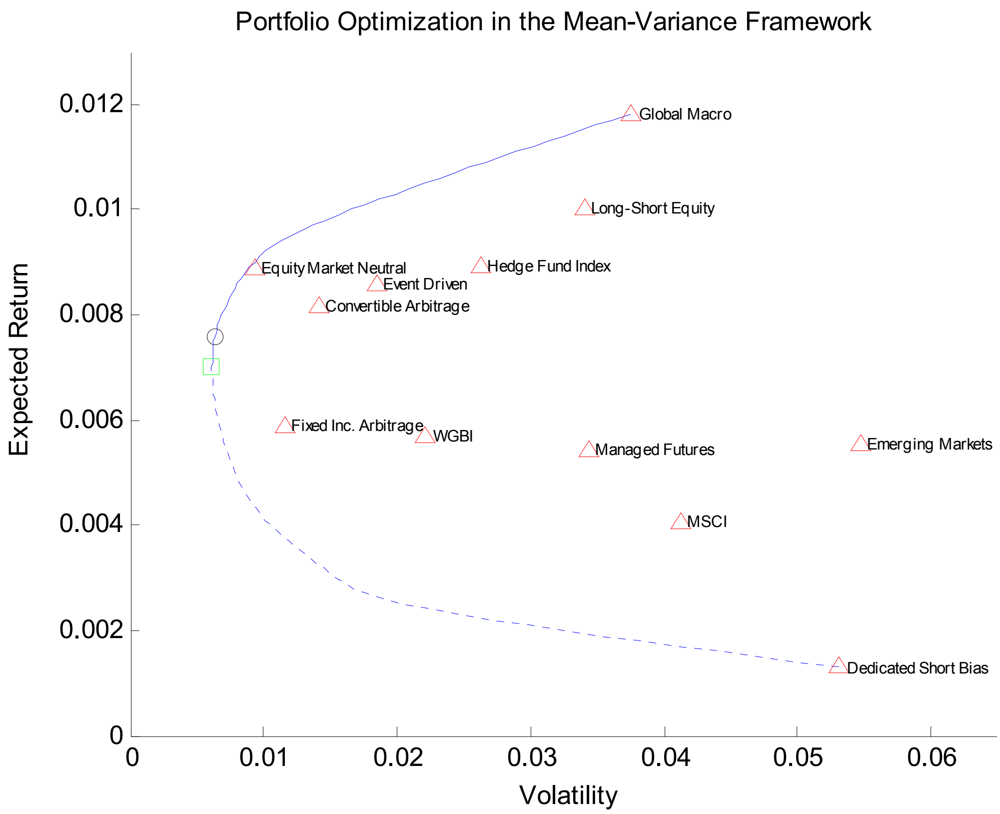


Figure 4.1: Minimum Standard Deviation Frontier. The solid line is the efficient frontier on which efficient portfolios are located. The dashed line is the inefficient part: any portfolio is dominated by (at least) one portfolio on the efficient set, i.e. for the same or a lower risk a better return can be achieved. The global minimum variance portfolio is represented by a square and the Sharpe ratio maximizing portfolio by a circle.

Mean-Variance analysis relies on the restrictive assumptions that either the investor's utility function is quadratic or the returns are normally distributed. It has been shown in section 2 that the quadratic utility function is inconsistent with the behavior of a rational investor. In addition, when asset returns are not normally distributed, mean and variance do not accurately capture the reward and risk characteristics of the distribution.

The main justification for the wide acceptance of the mean-variance analysis is its computational ease. However, the focus on the variance as the appropriate measure of risk implies that investors equally weigh the probability of below-the-mean returns and above-the-mean returns. Indeed, the variance is a measure of risk computed around the mean of the distribution. It is though very unlikely that investors' perception of downside risk is the same as the perception of upside potential. Related to this idea, the behavioral finance literature proposes the concept of loss aversion where individuals weigh losses more heavily than gains (see for instance Benartzi and Thaler (1995)). As suggested by Siegmann and Lucas (2002), there is evidence that loss aversion is an important notion to connect hedge funds particular pay-off structure and investors' utility function.

## 4.2 Mean - Downside Deviation Framework

An alternative formulation of the investment decision problem uses the downside deviation as a measure of risk. The downside deviation is an asymmetric measure of risk that focuses on the returns below a specified return target. The downside deviation is computed as follows:

$$downside\ \ deviation = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\tau - R_i)^2 \ \mathbf{1}_{R_i<\tau}}$$

where $R_i$ are the asset returns

$\tau$ is the return target

$n$ is the sample size

**1** is the indicator function.

Comparatively to the variance or standard deviation, the downside deviation has the advantage of not increasing with a greater upside potential. The information contained in the upside of the distribution does not contribute to the risk but is captured in the mean of the distribution.

The optimization problem is solved in the same manner as the traditional mean-variance framework, except for the definition of risk. The return target is set according to the investor's aversion to returns below a specific benchmark level. The mean-downside deviation approach is therefore more aligned with observed individuals' perception of returns, for which losses weigh more than gains. In addition, the framework uses less restrictive assumptions than mean-variance. It only requires general assumptions with respect to the investor's utility function, namely risk aversion and preference for skewness. In section 2 however, we have seen that investors also care about higher moments than skewness. Therefore, when returns are not normally distributed, this framework may be insufficient to fully capture investors' preferences.

## 4.3 Mean -Value-at-Risk Framework

Value-at-risk (VaR) reflects the potential downside risk faced by an investor in terms of nominal loss. Mainly used for both regulatory reporting and internal risk management purposes, VaR is merely a measure of the left tail of a returns distribution. Precisely, the measure is defined as the maximum probable loss on an investment over a specific period of time at a given confidence level. For a specific financial asset, the VaR can be read off the return cumulative distribution function. For monthly returns, the one-month VaR at a 99% confidence level corresponds to the value given at the 1% quantile of the distribution multiplied by the amount invested. In other words, 1% of the observed returns are lower than the VaR and 99% are higher.

Following Arzac and Bawa (1977), Huismann, Koedijk and Pownall (1999) develop a portfolio optimization model which allocates assets by maximizing the expected return subject to the constraint that the probable maximum loss meets the investor's VaR limit. In the framework, the risk is defined as the VaR relative to a benchmark return (for instance the risk-free rate). The authors argue that a mean-VaR approach fits with investors' behavior of minimizing exposure to large losses. The degree of risk aversion is reflected in the chosen VaR level and the associated confidence level. Broadly, the optimization process is similar to that of mean-variance except for the definition of risk.

Optimal allocation using empirical returns distributions does not require any assumption regarding the shape of the distributions. However, to accurately estimate the quantiles, the choice of the sampling period and the reliance on a large sample are essential. The main problem we face when assessing the empirical VaR of hedge fund returns is the scarcity and the low frequency of historical data. Accurate estimates of quantiles are thus not possible. An alternative is to assume a parametric distribution characterizing future returns. Usually, VaR is computed assuming normally distributed returns as follows:

$$VaR_{(1-\alpha)} = [\mu_R - z_c \sigma_R]\, W$$

where $W$ represents the size of the investment

$\alpha$ is the significance level (usually 1% or 5%)

$\mu_R$ is the expected return of the returns distribution

$\sigma_R$ is the standard deviation of the returns distribution

$z_c$ is the critical value of the normal standard distribution at a $(1-\alpha)$ threshold

In VaR calculation, it is customary to assume that the expected return over a short time period is zero. As we work with monthly data, we do not adhere to this assumption and include the mean return in our computation. It can also be noted that the size of the investment $W$ does not affect the maximization. Therefore, we set its value to one. Finally, because we are considering losses only, we take the absolute value of the VaR. Hence, assuming monthly normally distributed returns, our definition of a one-month 99% VaR is,

$$VaR_{99\%} = \left| \mu_R - 2.33 \times \sigma_R \right| .$$

Because the impact of the expected return is usually low, VaR is essentially a multiple of the standard deviation. As a consequence, under the assumption of normally distributed returns, the mean-variance and the mean-VaR frameworks lead to almost identical results and suffer from the same weaknesses. In particular, this approach is not appropriate for hedge fund returns. To overcome this problem, different models have been developed in the literature. For instance, Huismann, Koedijk and Pownall (1999) assume a student-t with 5 degrees of freedom distribution that more accurately captures the left tail of the distribution. Lhabitant (2001a) provides a model in which VaR is function of the risk exposure of hedge fund styles. Finally, Favre and Galeano (2000) propose a model in which the VaR is adjusted for skewness and kurtosis. Following the argumentation of the latter authors, we adjust the critical value of the normal standard distribution for skewness and kurtosis by using the Cornish-Fisher expansion:

$$z_{CF} = z_c + \frac{1}{6}(z_c^2 - 1)S + \frac{1}{24}(z_c^3 - 3z_c)K - \frac{1}{36}(2z_c^3 - 5z_c)S^2$$

where $z_c$ is the critical value of the normal standard distribution at a $(1-\alpha)$ threshold

$S$ is the skewness of the returns distribution

$K$ is the excess kurtosis.

Hence, the one-month 99% adjusted VaR is,

$$adjusted\ VaR_{99\%} = \left| \mu_R - z_{CF} \times \sigma_R \right|.$$

For the optimization problem, the risk/reward tradeoff faced by the investor is characterized in terms of adjusted VaR and expected return. The adjusted VaR does not assume normality and embodies the first four moments of the distribution. However, we have seen in section 2 that the investor's utility function is function of all the moments of the distribution. Hence this framework may not adequately characterize non normally distributed returns.

## 4.4 The need for a New Framework

Other models have been developed to deal with non-normally distributed returns. Berényi (2002) proposes a higher moment-based distributional risk measure, the variance-equivalent, which incorporates skewness and kurtosis. To justify the focus on the first four moments only, Berényi truncates the Taylor's series expansion of the investor's expected utility around its expected wealth after the fourth term. For non-normally distributed returns however, this reduction of the dimensionality requires two alternative assumptions. Either the investor's utility function is of polynomial type, which is not consistent with the behavior of a risk-averse investor for all levels of wealth, or it must be hypothesized that the truncated Taylor's series is a good approximation of the real expected utility function. Stutzer (2001) proposes an alternative performance index that is a generalization of the Sharpe ratio for returns distributions showing skewness and excess kurtosis. The model relies on the behavioral hypothesis of loss aversion and considers an investor who wishes to minimize the probability of receiving a negative time average excess return. When returns are not normally distributed, the index provides a preference parameter-free formula for the optimization problem that accounts for higher (positive) skewness and lower kurtosis preferences.

Frameworks assuming normality of returns are clearly inadequate for hedge fund portfolio analysis. More sophisticated models incorporating skewness and kurtosis fail to adequately represent investors' preferences for all the moments of the distribution when returns strongly depart from normality. Since hedge fund returns are usually far from normally distributed, portfolio analysis should be carried out taking into account all the moments of the distribution. As mentioned by Amin and Kat (2001) when assessing hedge fund performance over the last decade, “to evaluate the performance of portfolios with a non-normal, skewed returns distribution correctly, the entire distribution has to be considered. Ideally, this should be done without having to make any prior assumptions regarding the type of distribution”. We may add to the last sentence, that performance evaluation should be performed without assumptions regarding a risk-averse investor’s utility function too. The omega measure introduced by Keating and Shadwick (2002) provides a framework that exactly fits to these requirements.

# 5 A New Framework: The Omega Measure

## 5.1 Definition of the Function

Omega is a simple measure recently introduced by Keating and Shadwick (2002a, 2002b) which incorporates all the distributional characteristics of a returns series. The measure is a function of the return level and requires no parametric assumption on the distribution. Precisely, it considers the returns below and above a specific loss threshold and provides a ratio of total probability weighted losses and gains that fully describes the risk-reward properties of the distribution. It can therefore be used as a performance measure. The exact mathematical definition of the function is the following:

$$\Omega(r) = \frac{I_2(r)}{I_1(r)}$$

$$\text{where} \quad I_1(r) = \int_a^r F(x)\,dx \qquad \text{and} \qquad I_2(r) = \int_r^b (1 - F(x))\,dx$$

$F$ is the cumulative distribution function of the asset returns defined on the interval *[a, b]* and $r$ is the return level regarded as a loss threshold. For any investor, returns below its specific loss threshold are considered as losses and returns above as gains. A higher value of omega is always preferred to a lower value.

From a mathematical point of view, omega is equivalent to the returns distribution itself and embodies all its moments (see Cascon, Keating and Shadwick (2002) for a more detailed characterization of the mathematical properties of omega). The omega function is a unique monotone decreasing function of the cumulative distribution of returns from *[a, b]* to *[0, ∞[*. The function is differentiable and its first order derivative is thus always negative. On its domain of definition, the omega function of a risky distribution is flatter than that of a less risky distribution. The mean return of the distribution is the unique point at which the omega function takes the value one. At this point, the total probability weighted gains are equal to the total probability weighted losses. As stated by Keating and Shadwick (2002b), when the returns are normally distributed or when higher moments are insignificant, omega tends to agree with traditional measures such as the Sharpe ratio. However, as we illustrate it later, it may disagree and provide additional information for some return levels.

Figure 5.1 shows the omega functions defined on the interval [0, 2%] for hedge fund historical monthly returns provided by HFR (103 data points), namely Event Driven, Fund of Funds, Relative Value and Short Selling.

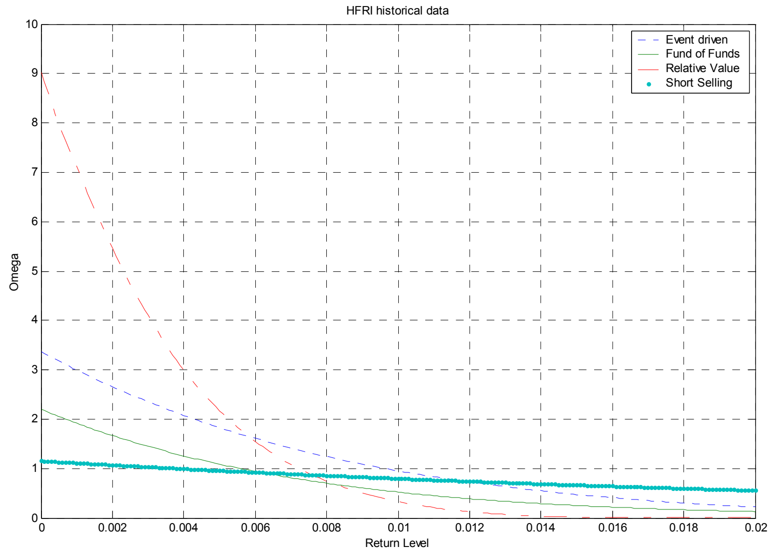


Figure 5.1: Omega Function for four indices of HFR historical data, Event driven, Fund of Funds, Relative Value, Short Selling. Omega is computed for a loss threshold varying between 0 and 2%.

Short Selling exhibits the flattest omega function and Relative Value the steepest, meaning that the returns distribution of the former is riskier than that of the latter. This fact is also reflected in their standard deviation, respectively 7.08% and 0.74%. Fund of Funds and Event Driven exhibit an intermediate position in terms of risk and the slope of their omega function makes it difficult to differentiate them, particularly at low return levels. As both curves tend to converge, Fund of Funds is nevertheless riskier than Event Driven. This is not verified by their respective standard deviation of 1.89% and 2.01%. Note therefore, that risk ordering based on standard deviation does not necessarily match the omega risk ordering. Indeed, as we show it later, any other moment of the distribution may have a determinant impact inducing a different order.

The crossing of the curves indicates a change of preference in the attractiveness of an asset. We see for instance that Relative Value is the most attractive asset (highest omega) at a zero return level. But for a loss threshold of approximately 0.6%, a risk averse investor is indifferent between Relative Value and Event Driven. For a loss threshold of 1%, Relative Value is the worst performing index. This example shows that asset ranking may vary depending on the threshold at which a return is regarded as a gain or a loss. For example, an investor may consider any return below the risk-free rate as a loss whereas another will only perceive losses for reductions of the initial capital. This contrasts with mean-variance which implies that investors equally consider below-the-mean returns and above-the mean returns.

We can show that even if returns are normally distributed, omega can add significant improvement to mean-variance analysis, by taking into account the investor's perception of loss and gain. Figure 5.2 displays the omega function for two HFR indices, Equity Hedge and Equity Market Neutral, which are normally distributed according to the Jarque-Bera test at a 99% confidence level. The two indices mostly differ by their first two moments; their respective Sharpe ratio for a risk-free rate of 0.1% is 0.40 and 0.69. According to this measure of attractiveness, Equity Market Neutral is always preferred to Equity Hedge. However, we observe in the graph that a change of preference occurs at a 0.54% loss threshold with omega. This means that for any investor whose perception of loss is set above this level, the ranking is reversed. This perfectly illustrates the additional information omega can bring even though returns are normally distributed.

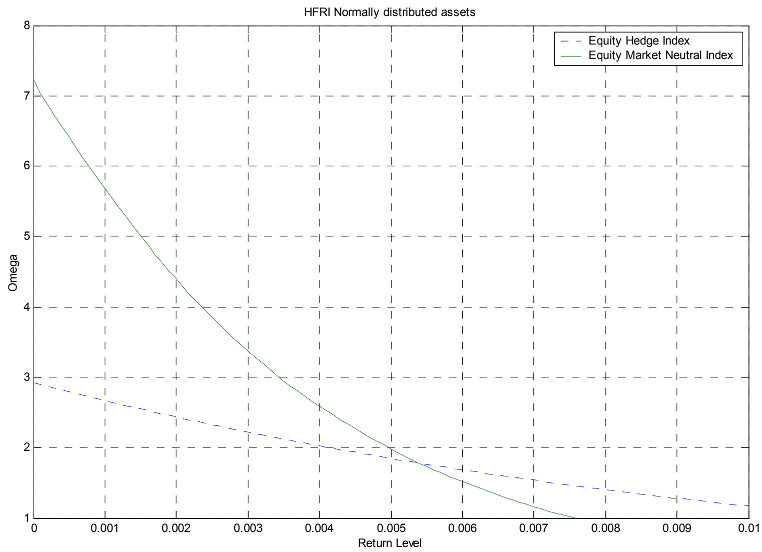


Figure 5.2: Omega function for two normally distributed indices, with a threshold varying between 0% and 1%

The loss threshold is function of the investor's preferences. For example, we may think that a risk averse investor confronted to investment selection will define ex-ante a higher threshold than a less risk averse individual. For the purpose of this paper, we always consider the loss threshold as exogenously defined.

Omega agrees with the investor's perception of downside and upside and provides a risk-reward measure weighted by their probability of occurrence. Basically, omega requires no specification of a utility function for asset ranking and suits any risk-averse investor; all we just need to consider is that more money is preferred to less money. The omega function can nevertheless be accommodated with any utility function characterizing an investor. As shown in section 2, a risk-averse investor has a strictly concave and increasing utility function meaning that he never displays satiation. Any monotone increasing function can therefore be used as a transformation of the returns and will induce local changes of preference in the omega function.

In the previous sections, we argue that investors care about higher moments and are more averse to losses than gains. This suggests that a lower partial moments measure incorporating all the moments of the distribution should represent a more appropriate definition of risk. Such a measure is represented in the omega framework by the total probability weighted losses $I_1(r)$.

The omega measure provides a major contribution at different levels. First, omega incorporates all the moments of the distribution and is therefore appropriate for investment analysis when returns are not normally distributed. Second, even for normally distributed returns, omega provides additional information since it takes into account the investor's preferences for loss and gain. Finally, omega is computed directly from the returns distribution and measures the total impact of the moments instead of each one of them individually. It can therefore reduce the estimation error risk[2].

## 5.2 Influence of Higher Moments

Many authors in the literature (see for example Brooks and Kat (2001), Favre and Galeano (2000) and Berenyi (2002)) define the risk with a measure including not only the variance but also skewness and kurtosis, implying that negative skewness and excess kurtosis increase the risk of a distribution. According to these measures, we should expect a normal distribution to be more attractive (because of its lower risk) than any other distribution with the same mean and variance but exhibiting negative skewness and/or excess kurtosis. In order to check if this is verified in the omega framework, we mix normal distributions to simulate four distributions of 200 data points each, with different statistical properties. They all have the same mean and standard deviation, respectively 0.8% and 3%. The first distribution is normal, the second has a negative skewness of -1, the third one is leptokurtic with a kurtosis of 6 and the last one has both negative skewness (-1) and high kurtosis (6). We expect the negatively skewed and the leptokurtic distributions to be less attractive (because of their higher risk) than the normal. The distribution exhibiting both negative skewness and high kurtosis, which is the closest to a typical hedge fund index returns series, should be the least attractive among the four simulated distributions. Figure 5.3 displays the omega function for the simulated distributions.

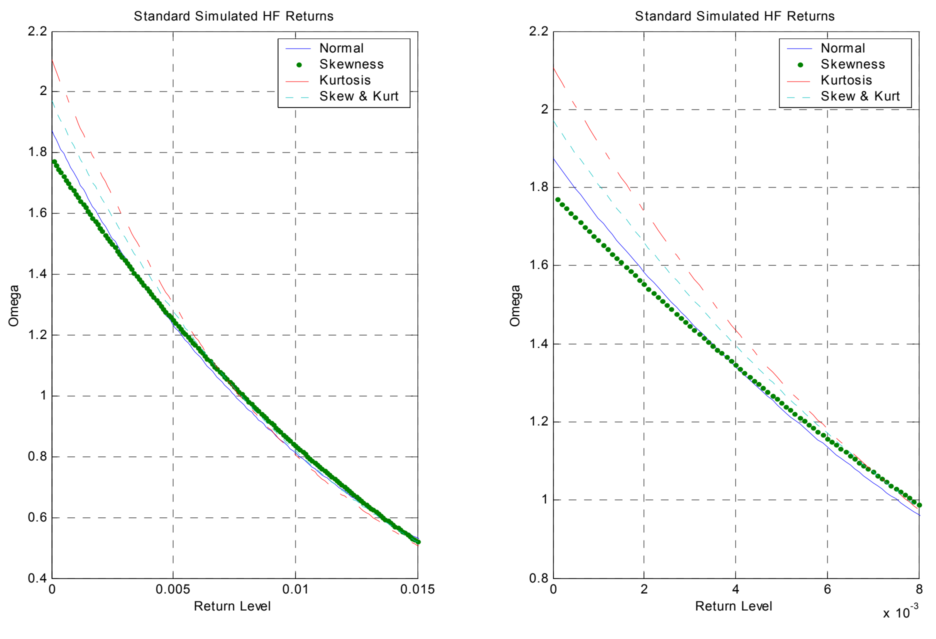


Figure 5.3: Omega function for four simulated distributions, with a threshold between 0% and, respectively, 1.5% on the left and 0.8% on the right.

The graphs above clearly illustrate the impact of higher moments than kurtosis on the attractiveness of an asset. Focusing for instance on the right graph at the loss threshold of 0.2%, it is particularly evident that comparatively to the normal distribution, only the distribution showing negative skewness is less attractive. On

[2] The drawback is that it makes very difficult to exactly distinguish which moments matter.

the contrary, the leptokurtic as well as the skewed and leptokurtic distributions exhibit a higher omega. By comparing the non-normal distributions between them, the skewed and leptokurtic distribution should exhibit the lowest attractiveness. We observe however that the lowest omega is displayed by the negatively skewed distribution. These results suggest that models focusing on the first four moments only, may not always provide an adequate measure of attractiveness for non-normally distributed returns

We can show with historical data that the inclusion of all the moments in the analysis improves the performance measure compared to the Sortino ratio and the adjusted VaR modified Sharpe ratio (see appendix 2 for the mathematical formulation of these measures). Figure 5.4 shows the adjusted-VaR modified Sharpe ratio, the Sortino ratio and the omega function for four HFR hedge fund indices (Equity Hedge, Equity Market Neutral, Merger Arbitrage, Relative Value) at different return levels. Each time series contains 103 data points. Equity Hedge and Equity Market Neutral are normally distributed according to the Jarque-Bera statistics at a 99% confidence level. Merger Arbitrage and Relative Value are far from normality with high negative skewness and excess kurtosis.

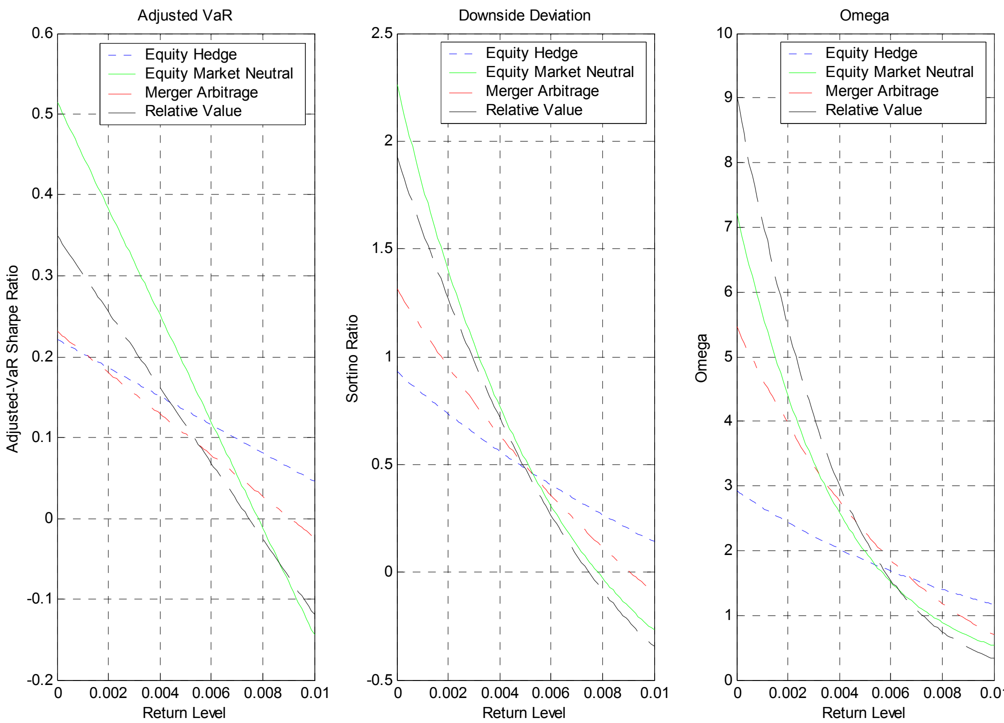


Figure 5.4: Adjusted VaR Modified Sharpe ratio, Sortino ratio and omega function for four HFR hedge fund indices (Equity Hedge, Equity Market Neutral, Merger Arbitrage, Relative Value) with a return level varying between 0 and 1%.

As the adjusted VaR Modified Sharpe ratio and the Sortino ratio only take into account a limited number of moments, the discrepancies between these measures and omega illustrate the importance of including all the moments in the analysis. For instance, at 0.2% and 0.4% return levels, we note that the adjusted-VaR modified Sharpe ratio and omega provide totally different rankings in terms of performance, while the discrepancies are less numerous between the Sortino ratio and omega. At a 0.8% threshold, all the measures agree.

Clearly, measures truncating Taylor's series expansion are not satisfactory. To better assess the implications of such a truncation, we simulate two sets of data based on the statistical properties of historical hedge fund returns provided by CSFB and HFR. First, by using the technique of the generalized method of moments, we combine 3 normal distributions to replicate for each index a returns distribution with the same first eight moments (simulations A). Then, we simulate distributions (simulations B) that only replicate the first four moments of the historical distribution; the moments of order 5 to 8 are those of a normal distribution[3]. Each series contains 200 data points (versus 103 for historical data), which has the advantage to overcome the problem of scarcity of data in the hedge fund industry.

Figure 5.5 shows the omega function for three indices drawn from HFR simulations A and simulations B, Distressed Securities, Equity Non Hedge and Event Driven indices. Only Equity Non Hedge is normally distributed according to the Jarque-Bera (95%).

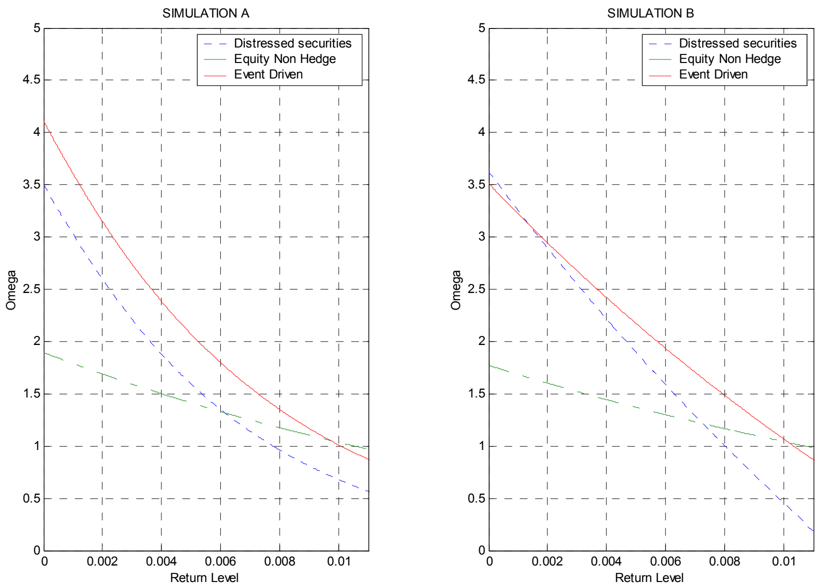


Figure 5.5: Omega function for three indices drawn from HFR simulations A (left) and simulations B (right), Distressed Securities, Equity Non Hedge and Event Driven, with a return threshold between 0% and 0.8%.

First, we observe that the omega functions of the two non-normally distributed indices exhibit a different shape between simulations A and simulations B. But the omega function of Equity Non Hedge tends to remain unchanged in both cases since it is normally distributed. Second, the comparison between the two graphs gives rise to markedly different rankings. On the left, a change of preference between Distressed Securities and Equity Non Hedge appears at a 0.6% threshold while on the right, where moments of order five and higher are

[3] Note however that by setting the moments of order 5 to 8 at the level of a normal distribution we also indirectly change the moments of order 9 and higher. As Omega incorporates all the moments of the distribution, these high moments will exhibit important differences comparatively to a normal distribution and will impact the omega function. Therefore, we are not able to exactly replicate the truncation of the Taylor's series expansion. However, it can give us an interesting insight of the effect of a modification of higher moments.

modified, it happens at a 0.73% threshold. Between Distressed Securities and Event Driven, in the left graph, the ranking remains the same in the whole range of thresholds, whereas in the right graph at 0.15% there is a change of ranking. The modification of moments of higher order than four clearly has an impact on the omega function. It thus suggests that measures truncating the Taylor's expansion cannot provide a reliable image of the distribution.

## 5.3 Asset Performance Ranking

In this section we analyze hedge fund historical performance ranking with different measures, the Sharpe ratio, the VaR modified Sharpe ratio, the adjusted-VaR modified Sharpe ratio, the Sortino ratio and omega. Our approach involves three steps. First, we compare the rankings provided by all the measures, using a risk-free rate 0.1%, which is the 3-month Libor monthly rate as of July 2002. To compute the downside deviation and omega values, we also use a loss threshold of 0.1%. Secondly, we analyze the Sortino and omega rankings at three different return levels: 0%, which means that any reduction of the initial investment is regarded as a loss, 0.l% (risk-free rate) and 0.63%, which is the mean return of the HFR Fund of Funds index and can be seen as a benchmark in the hedge fund industry. Finally, we compare the omega rankings at 0.1% and 0.63% fixed thresholds with time corresponding synchronized thresholds. To do this, from a hedge fund index and a benchmark time series, we compute a new series of hedge fund index returns in excess of the time varying benchmark. Then, we calculate omega at a zero loss threshold.

Appendix 3 shows the various rankings for the two sets of historical returns. Panel A exhibits the rankings for CSFB-Tremont indices as well as WGBI and MSCI. Panel B displays the HFR indices with the same traditional indices. Not surprisingly, for a risk-free rate and a return level of 0.1%, we note that the two measures assuming normality (Sharpe ratio and VaR modified Sharpe ratio) exhibit no difference in the ranking of the two sets of indices. This is because the VaR is, according to our definition, essentially a multiple of the standard deviation (see section 4.3). When we contrast the rankings of these two measures with those of adjusted VaR modified Sharpe ratio, Sortino ratio and omega, we find that they are markedly different. For instance, the two CSFB-Tremont indices that most depart from normality, Event Driven and Fixed Income Arbitrage, see their relative position change with measures taking into account higher moments. It can also be observed that the adjusted VaR modified Sharpe ratio rankings tend to be quite different from those of Sortino and omega.

Focusing on the Sortino and omega rankings at the different thresholds, we see that each measure exhibits little change between the 0% and the 0.1% thresholds. This is not a surprise since the return levels are very close and few returns are affected by the change of threshold. The converse can be observed relatively to the 0.63% threshold that exhibits very different rankings. If we compare the Sortino and omega rankings at the same threshold, we find that the differences are less marked for higher threshold. This suggests that the impact of moments of order 5 and higher lessens with higher thresholds.

Regarding fixed versus time varying thresholds, we observe that the rankings with the risk-free rate thresholds exhibit no or little difference for both CSFB-Tremont and HFR indices. Indeed, the Libor has a very low variation over time (volatility of 0.08%). On the contrary, the fixed 0.63% and the synchronized Fund of Funds thresholds display markedly different rankings as the Fund of Funds index exhibits more important variations over time (volatility of 1.89%).

Globally, hedge fund performance ranking can be broadly split in two groups of measures: on the one hand, measures assuming normality of returns such as Sharpe and VaR modified Sharpe ratios; on the other hand omega, Sortino and adjusted VaR modified Sharpe ratios. This indicates that omega, like downside deviation and adjusted VaR, is noticeably affected by the first four moments of the distribution. However, omega tends to exhibit different results, highlighting the fact that moments of order 5 and higher should be included in the analysis. But because omega measures the global influence of all the moments, it is not possible to determine exactly which moments impact most.

## 5.4 Sensitivity Analysis to Sample Size

In order to check the sensitivity of omega relatively to the sample size, we perform simulations of typical hedge fund monthly returns series with the following characteristics: mean 0.8%, standard deviation 3%, skewness -1 and kurtosis 6. By varying the sample size from 36 to 600 data points, we simulate 20 returns time series for each sample size and compute their omega values at three different thresholds: 0%, 0.4% and 0.8%. Note that the latter threshold corresponds to the mean of the distributions. Then, we calculate the average value of the omegas for each sample size. Figure 5.6 plots the average omega versus the sample size. It can be observed that the omegas converge toward their theoretical value. In particular, for a threshold of 0.8%, omega converges toward one. We find that sample sizes below 100 data points show important variation whereas after 200 data points, higher sample sizes do not provide important improvement.

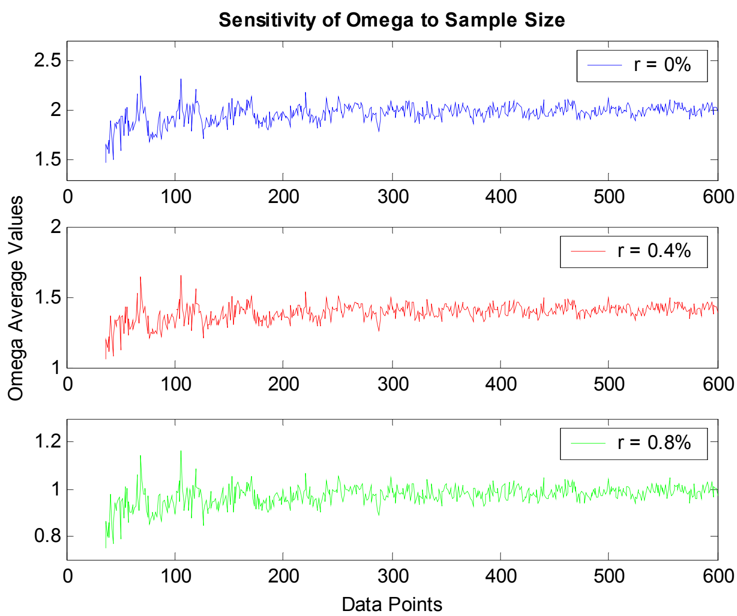


Figure 5.6: Sensitivity of omega to Sample Size for different loss thresholds. Typical hedge fund returns series with negative skewness and excess kurtosis are simulated with a sample size varying from 36 data points to 600 data points.

For each sample size, we also compute the standard error of omega around its true value. This leads to figure 5.7 where we can see the impact of larger sample sizes on the reduction of the estimation error. It is particularly obvious that the uncertainty linked to the estimation of omega tends to reach a floor after 200 data points. We can therefore argue that a time series should include at least 100 observations to give consistent results with omega. However, more accurate results should be obtained with 200 data points. This corresponds to around 17 years of historical monthly returns.

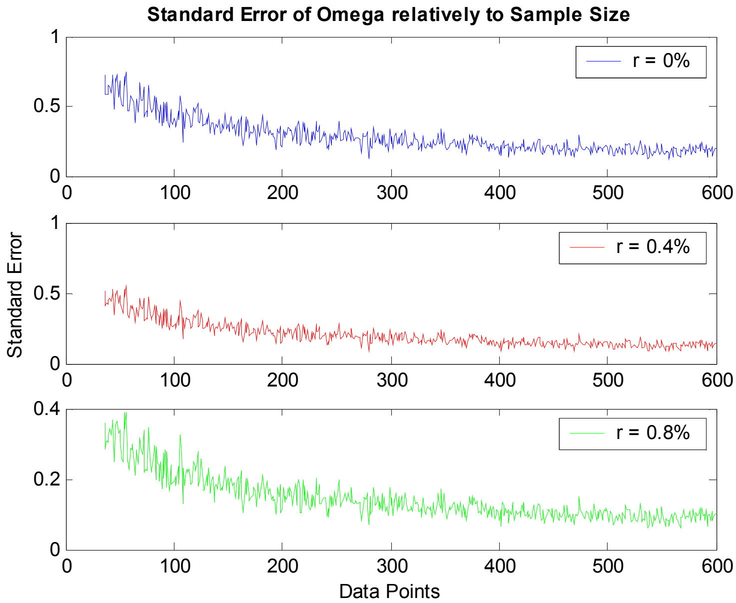


Figure 5.7: Standard Error of omega around its true value for different loss thresholds, with respect to sample size. Typical hedge fund returns series with negative skewness and excess kurtosis are simulated with a sample size varying from 36 data points to 600 data points.

A natural question at this stage is whether the omega measure is reasonably accurate or not. To answer this, we empirically compute a 95% pseudo-confidence interval. For returns with the same statistical features as earlier, we simulate 1000 distributions of 200 data points each and compute their omega value at a loss threshold of 0.8%. We find an average omega of 0.98, which is close to its theoretical value of one[4]. The 2.5% and 97.5% quantiles of the omega distribution give values of respectively 0.71 and 1.31. Therefore, omega's 95% relative confidence interval around the mean is approximately [-28%, 34%]. As it can be seen, the omegas are not symmetrically distributed. This suggests that for samples of 200 data points, omega is more likely to be overestimated than underestimated. Focusing separately on the numerator and the denominator of omega, we find a 95% relative confidence interval of [-19%, 19%] for the total probability weighted gains ($I_2$) and of [-17%, 19%] for the total probability weighted losses ($I_1$). The relative confidence intervals computed for other thresholds show similar results.

[4] Note that the theoretical value cannot be accurately achieved with a small sample size because of the approximation of omega in the numerical integration of a discrete returns distribution.

We find it interesting to contrast the omega confidence interval with that of the Sharpe ratio. We thus compute the ratio on the same simulated distributions. We find that the Sharpe ratio exhibits a greater confidence interval than omega. For a risk-free rate of 0.1%, the average Sharpe ratio is 0.23 and the 2.5% and 97.5% quantiles are 0.12 and 0.35. The resulting relative confidence interval around the mean is about [-50%, 50%]. With higher risk-free rates, the interval increases. The results for the Sortino ratio are similar. By observing separately the variation of the expected return estimator and the risk estimator for the two measures, we find that the high variation of the ratios is due in a large part to the expected return. Indeed, the 95% confidence interval of the mean return for both measures is [-48%, 48%]. On the contrary, the variance or downside deviation estimator tends to converge much quicker to the true value for a sample size of 200. Their 95% confidence interval is respectively [-8%, 7%] and [-8%, 10%]. This reflects the well known problem of large estimation error in the estimated expected returns documented in the literature (see for instance Jorion (1985)) that makes expected returns very difficult to estimate. Omega may therefore provide a method to reduce the estimation risk in portfolio analysis.

## 5.5 Sensitivity Analysis to Outliers

To evaluate the impact of extreme returns on the omega function, we remove the 2.5% and 97.5% quantiles from the returns distributions. As most of hedge fund indices show negative skewness and high kurtosis properties, we can expect the downside to be dramatically improved and the upside to be negatively affected but in a smaller extent. We can thus anticipate an increase of the values of omega.

We show the impact of the stress testing on two different indices, CSFB-Tremont Event Driven and HFR Convertible arbitrage. Figure 5.8 exhibit the histogram of CSFB-Tremont Event Driven before and after the stress testing. As shown in the left graph, the historical returns are far from normality. We particularly remark the single extreme loss that occurred in August 1998 because of the Russian default crisis (-11.8%). Once the extreme returns are removed, the histogram on the right shows that variance, skewness and kurtosis dramatically improve and the distribution is normal according to the 95% Jarque-Bera statistic. Because the outliers are not symmetrically distributed, the mean return increases too.

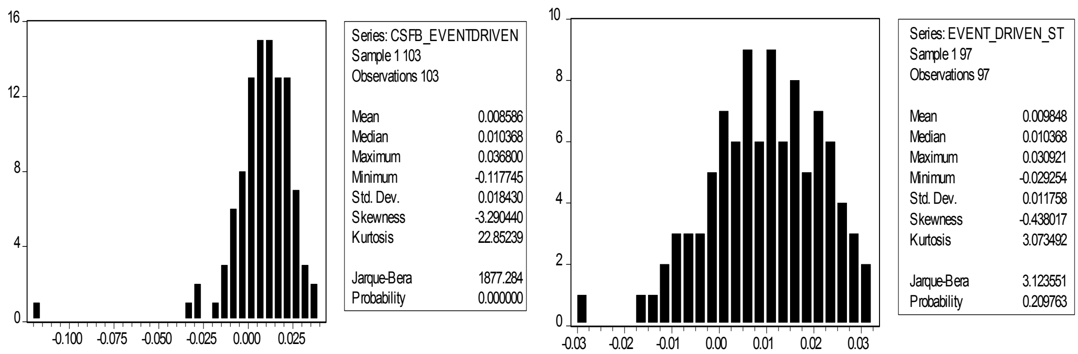


Figure 5.8: histogram of the returns distribution of CSFB-Tremont Event Driven before (left) and after (right) elimination of the 5% extreme returns

Figure 5.9 present the histograms of HFR Convertible Arbitrage before and after the stress testing. In the left graph, we also remark extreme losses that occurred in November 1998, in the aftermath of the Russian crisis and the LTCM collapse. With the stress testing, we broadly obtain the same changes as above for the variance, the skewness and the kurtosis. However, the rise of the mean return is lower since the historical returns distribution is more symmetric than CSFB-Tremont Event Driven. Note also that the Jarque-Bera test still rejects normality for HFR Convertible Arbitrage after the stress testing.

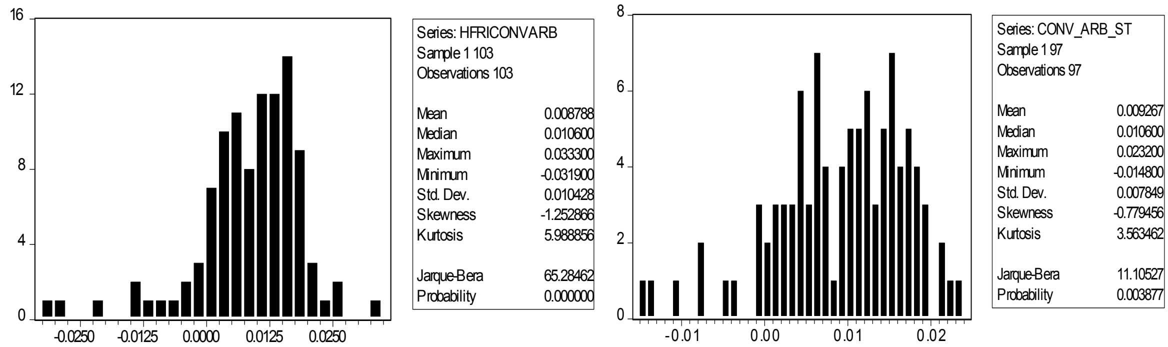


Figure 5.9: histogram of the returns distribution of HFR Convertible Arbitrage Index before (left) and after (right) elimination of the 5% extreme returns

Figure 5.10 shows the impact of the stress testing on the values of omega. As expected, removing extreme returns has a significant impact on the omega function, both in terms of risk of the distribution (steeper slope) and attractiveness (higher values). For higher thresholds, the downside measure is less affected by the stress testing (the mass of losses is greater, therefore a lower proportion is removed) and the upside is more affected. That is why the two curves tend to converge.

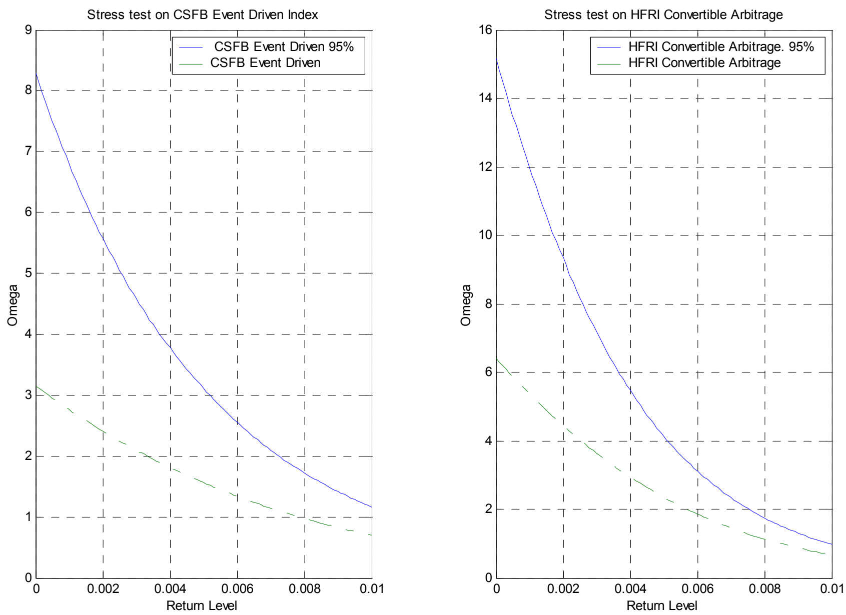


Figure 5.10: Omega function of CSFB-Tremont Event Driven (left) and HFR Convertible Arbitrage Index (right) before and after elimination of the 5% extreme returns

# 6 Portfolio Optimization in the Omega Framework

The new approach developed by Keating and Shadwick (2002a) provides a useful tool to compare and rank assets according to their statistical characteristics. However, it offers limited insight of the potential of omega for optimal asset allocation. In this chapter, we investigate the empirical properties of the omega measure as a portfolio optimization tool.

## 6.1 Methodology

To perform portfolio optimization, we use the historical data sets described in section 3.2, namely the hedge fund indices provided by CSFB-Tremont and HFR as well as the traditional asset class indices represented by WGBI and MSCI. We perform asset allocation in three different investment universes. First, we consider portfolios of asset class indices, combining bonds, equity and a hedge fund class index. Then, we form portfolios of hedge fund style indices only. Finally, we analyze portfolios mixing traditional asset class indices, hedge fund style indices and hedge fund composite indices. The constituted portfolios are the following:

Portfolios combining alternative investment and traditional asset classes:

- **Portfolio 1 (Global HFI)** is invested in the CSFB-Tremont composite Hedge Fund Index, WGBI and MSCI.
- **Portfolio 2 (Global FoF)** comprises the HFR Fund of Funds, WGBI and MSCI.

Portfolios of hedge fund styles:

- **Portfolio 3 (Hedge Funds CSFB)** is invested in the 9 CSFB-Tremont strategy sub-indices.
- **Portfolio 4 (Hedge Funds HFR)** contains 14 HFR strategy sub-indices (Fund of Funds and Fixed Income (Total) are not included).

Portfolios mixing hedge fund style and composite indices with traditional asset classes:

- **Portfolio 5 (Global CSFB)** includes the 9 CSFB-Tremont sub-indices as well as WGBI and MSCI.
- **Portfolio 6 (Global HFR)** includes the 16 HFR indices, WGBI and MSCI.

Throughout this section, we contrast the results obtained in the omega framework with other commonly used settings, namely mean-variance, mean-Value-at-Risk, mean-adjusted Value-at-Risk and mean-downside deviation. Appendix 2 summarizes the different measures of risk and reward used in each framework. Basically, the same methodology is applied in each framework to derive an efficient frontier. The weights of the indices are combined so that an optimal portfolio is formed with minimum risk for a specific reward. In order to be consistent with the reality, we impose a non-negativity constraint on the weights of the portfolios. For the omega and the mean-downside deviation frameworks, we keep the loss thresholds defined earlier for the performance ranking (section 5.3), i.e. 0%, 0.1% and 0.63%. In the omega framework, portfolio optimization is also performed with synchronized loss thresholds (time varying benchmark).

Two optimal portfolios are of special interest because they allow a fair comparison between the different settings. The first one is the global minimum risk portfolio which is located at the extreme left of the efficient frontier in a traditional risk/reward representation. The second portfolio is the performance maximizing portfolio which may be located higher on the efficient set. It is well known however, that the practical application of portfolio analysis in the commonly used frameworks may be seriously hampered by the expected returns estimation error (see section 5.4). The global minimum risk portfolios do not suffer from this bias since they are the only portfolios independent of the return level. Therefore, in the following sections, we mainly focus on the characteristics of the global minimum risk portfolios.[5]

## 6.2 Empirical Results

In the next three sections, we discuss the results obtained in the different frameworks, with a zero loss threshold for the mean-downside deviation and the omega frameworks. The zero threshold is a natural choice implied by the fact that mean-variance, mean VaR and mean-adjusted VaR efficient frontiers are derived without risk-free asset. Section 6.2.4 focuses on the omega and mean-downside deviation settings to show the impact of higher thresholds on optimal allocation. In section 6.2.5, we discuss the results obtained with omega for time varying thresholds. Finally, section 6.2.6 shows evidence regarding the efficiency of the different frameworks when returns are not normally distributed.

### 6.2.1 Global Asset Classes Universe

It is well-known that mean-variance analysis is not appropriate for hedge fund returns. For instance, Brooks and Kat (2001) argue that mean-variance tends to overestimate the true risk-return performance of hedge funds, which leads to over-allocate to this asset class. Comparatively to settings with no distributional assumption, we might therefore expect that frameworks assuming normality overweight hedge funds. Table 6.1, which contains the weights of the global minimum risk portfolios in the different frameworks, tends to moderate this argumentation, particularly with respect to the omega optimization.

| Global Minimum Risk Portfolios | | | | | |
|---|---|---|---|---|---|
| | Mean-Variance | Mean-VaR | Mean-adusted VaR | Mean-D.Dev. 0% | Omega 0% |
| **Portfolio 1 : Global Hedge Fund Index** | | | | | |
| Hedge Fund Index | 41% | 44% | 44% | 44% | 50% |
| SSB WGBI | 58% | 56% | 56% | 55% | 49% |
| MSCI | 1% | - | - | 1% | 1% |
| **Portfolio 2 : Global Fund of Funds** | | | | | |
| Fund of Funds | 54% | 54% | 41% | 53% | 70% |
| SSB WGBI | 46% | 46% | 57% | 47% | 30% |
| MSCI | 0% | 0% | 2% | - | - |

Table 6.1: Global Minimum Risk Portfolios obtained in different frameworks. Each portfolio combines a hedge fund class index with traditional asset classes.

[5] It can be noted anyway that optimization for both portfolios tend to behave accordingly. Therefore, the conclusions are broadly the same.

First, we observe that an important place is given to the hedge fund class with weights up to 50% for Portfolio 1 and 70% for Portfolio 2. Conversely, MSCI is given very little or no weight. The reason is that in any framework, the equity index has the lowest reward for the highest risk. It is obvious that MSCI is considerably affected by the recent financial market turmoil. Under normal market conditions or with historical data over a longer period, we might expect MSCI to be given a significant share in the portfolios.

Focusing on Portfolio 1, we note first that its three constituents are normally distributed according to the Jarque-Bera statistics at a 95% confidence level. Therefore, any combination of these assets results in a portfolio with normally distributed returns. In theory, all the frameworks should give identical results but this is not exactly the case because the indices are not perfectly normal. We see for instance that omega invests the largest fraction in Hedge Fund Index (HFI) whereas the other frameworks place WGBI as main asset. This actually reflects a change in risk ordering among the indices (see appendix 4 for the risk ordering of all the indices). HFI is considered as the least risky index in the omega setting but is ranked second in all the other frameworks. This is clearly an effect of moments of higher order than kurtosis. The impact of higher moments can be analyzed from the portfolios perspective. Table 6.2 shows the statistical properties of the global minimum risk portfolios. We can see for Portfolio 1 that its standard deviation, skewness and kurtosis properties for the various settings are almost identical. For the mean return and moments of order 5 to 8 however, the omega optimization shows different figures. Moments 5 and 7 are lower than the other settings and moments 6 and 8 are higher. In theory, their effect on the portfolio attractiveness is adverse, but if we compare the values of omega, the converse occurs. The higher attractiveness of the portfolio derived in the omega framework comes in fact mostly from the higher reward ($I_2$) induced by the higher mean return.

| **Global Minimum Risk Portfolios** | | | | | |
|---|---|---|---|---|---|
| Statistics | Mean-Variance | Mean-VaR | Mean-adusted VaR | Mean-D.Dev. 0% | Omega 0% |
| **Portfolio 1 : Global Hedge Fund Index** | | | | | |
| Mean | 0.70% | 0.71% | 0.71% | 0.71% | 0.73% |
| Standard Dev. | 1.66% | 1.66% | 1.66% | 1.66% | 1.69% |
| Skewness | -0.1 | -0.1 | -0.1 | -0.1 | -0.1 |
| Kurtosis | 2.8 | 2.8 | 2.8 | 2.8 | 2.9 |
| 5th moment | -0.1 | -0.2 | -0.2 | -0.3 | -0.7 |
| 6th moment | 11.4 | 11.2 | 11.2 | 11.3 | 12.4 |
| 7th moment | 1.7 | 0.3 | 0.3 | -1.0 | -7.2 |
| 8th moment | 56.9 | 53.9 | 53.9 | 55.0 | 65.5 |
| Omega 0% | 2.68 | 2.72 | 2.72 | 2.72 | 2.77 |
| $I_1$ | 0.393% | 0.391% | 0.391% | 0.391% | 0.390% |
| $I_2$ | 1.053% | 1.062% | 1.062% | 1.061% | 1.078% |
| **Portfolio 2 : Global Fund of Funds** | | | | | |
| Mean | 0.60% | 0.60% | 0.59% | 0.60% | 0.61% |
| Standard Dev. | 1.40% | 1.40% | 1.47% | 1.40% | 1.44% |
| Skewness | -0.1 | -0.1 | 0.0 | -0.1 | -0.3 |
| Kurtosis | 3.5 | 3.5 | 3.2 | 3.4 | 4.8 |
| 5th moment | -2.9 | -2.9 | 0.5 | -2.3 | -8.5 |
| 6th moment | 23.5 | 23.5 | 17.8 | 21.9 | 51.2 |
| 7th moment | -39.2 | -39.2 | 8.5 | -30.3 | -161.3 |
| 8th moment | 206.9 | 206.9 | 126.9 | 181.2 | 739.0 |
| Omega 0% | 2.79 | 2.79 | 2.61 | 2.78 | 2.82 |
| $I_1$ | 0.313% | 0.313% | 0.341% | 0.314% | 0.310% |
| $I_2$ | 0.872% | 0.872% | 0.891% | 0.873% | 0.874% |

Table 6.2: Statistical Properties of global minimum risk portfolios

If we focus on Portfolio 2, we see in table 6.1 that the weight of Fund of Funds (exhibiting a skewness of -0.2 and a kurtosis of 6.2) in the omega setting is much higher than that of WGBI (respectively 70% and 30%). In the other frameworks, the differences are smaller. We can see in table 6.2 that moments of order 5 to 8 for the omega optimization markedly depart from those of the other settings. The improved attractiveness of the portfolio in terms of omega values is mainly due to the reduced risk ($I_1$). But in this case, it is very difficult to determine which moments impact most on the risk improvement effect. Indeed, it can be induced by any higher moment than the eighth one or result from the combined effect of all the moments of the distribution,

It may seem surprising that Portfolio 2 places a weight of 70% on Fund of Funds (FoF) while Portfolio 1 invests only 50% in HFI. In fact, it can be seen for Portfolio 2 that the mean return of the global minimum risk portfolio derived in the omega setting is 0.61% versus 0.59% or 0.60% for the other settings. FoF displays a slightly higher mean return with 0.63% but WGBI and MSCI are much lower with only 0.57% and 0.40%. Because of the non-negativity constraint, there is almost no room for FoF to compensate for the poor contribution of WGBI and MSCI in terms of reward. To include a larger portion of traditional indices, the global minimum risk portfolio would require a hedge fund index with high reward enhancement features, which is not the case of FoF. Therefore, at this level of expected return, the global minimum risk portfolio is forced to place a high weight on FoF and a slight change in the expected return of the optimal portfolio induces an important increase of the weight of FoF.[6] In the case of Portfolio 1, the mean return of HFI is 0.89%, which is much higher than that of the global minimum risk portfolio derived in the omega framework (0.73%). In this case, HFI provides high reward enhancement characteristics and allows the inclusion of a larger portion of WGBI.

All this may confirm that a well diversified investment in hedge funds can add valuable risk diversification and performance enhancement to a portfolio of traditional assets. Because omega provides a more appropriate definition of risk and reward, we can argue that the contribution of hedge funds is greater than what is suggested by (less accurate) traditional measures. This would provide an explanation for the fact that omega increases the weight of the hedge fund class. However, we still need to be careful about the interpretation of the weights since under normal market conditions, the equity asset class would certainly gain significant weight and impact the allocation to hedge funds.

### 6.2.2 Hedge Fund Styles Universe

With many assets in a portfolio, the interrelations between indices (co-moments) and their effect on the optimization makes the results much more difficult to explain. In this section, we try to show the major trends contrasting the different frameworks. Appendix 5 (panel A) shows the basic statistics relative to the global minimum risk portfolios for Portfolios 3 and 4. The weights of Portfolio 3 (Hedge Funds CSFB) are shown in table 6.3.

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[6] If we take the extreme case where the global minimum risk portfolio has an expected return of 0.63%, 100% of the weight would be placed on FoF. Thus, a rise of two basis points in the expected return (from 0.61% to 0.63%) of the optimal portfolio would mean a jump of 30 percentage points in the weight of FoF (from 70% to 100%).

| Global Minimum Risk Portfolios | | | | | |
|---|---|---|---|---|---|
| | Mean-Variance | Mean-VaR | Mean-adusted VaR | Mean-D.Dev. 0% | Omega 0% |
| **Portfolio 3 : Hedge Funds CSFB** | | | | | |
| Convertible Arbitrage | - | - | - | - | - |
| Dedicated Short Bias | 9% | 9% | 11% | 13% | 11% |
| Emerging Markets | - | - | 4% | 4% | 4% |
| Equity Market Neutral | 55% | 61% | 61% | 59% | 62% |
| Event Driven | 2% | 3% | - | - | 1% |
| Fixed Income Arbitrage | 28% | 22% | 12% | 11% | 12% |
| Global Macro | - | - | - | - | - |
| Long-Short Equity | 6% | 6% | 11% | 13% | 9% |
| Managed Futures | 0% | - | 1% | 0% | 2% |

Table 6.3: Global Minimum Risk Portfolios of CSFB-Tremont hedge fund style indices derived in different frameworks.

Globally, we note for Portfolio 3 that the optimizations performed in the different frameworks do not give rise to very important discrepancies. This is because the risk ranking of Portfolio 3 tends to be quite stable. As a general rule we can say that the optimal portfolios invest predominantly in indices with lower risk. For example, we observe in every setting the predominant position of Equity Market Neutral (between 55% and 61%), which is the index showing the lowest risk among the different measures. Fixed Income Arbitrage (second or third position in terms of lower risk) and Long Short Equity (fifth position) are also given significant weight. It can be noticed however, that Convertible Arbitrage (that always swaps its second or third position with Fixed Income Arbitrage) is never included in the portfolio. Moreover, a significant weight is usually placed on Dedicated Short Bias despite its very high risk. The reason is that it is negatively correlated with the other indices and can therefore contribute to a large extent to risk diversification. Finally, the last four strategies, Emerging Markets, Event Driven, Global Macro and Managed Futures have little or no weighting. They all tend to suffer from high risk. By contrasting the frameworks assuming normality (mean-variance and mean-VaR) with the other frameworks with no distributional assumption, we find that the greatest disparity concerns Fixed Income Arbitrage. The reason is that among the indices included in Portfolio 3 with significant weighting, it is the only one showing negative skewness and important leptokurtic properties. Because its true risk is underestimated in the mean-variance and mean-VaR settings, it is therefore over-weighted.

Table 6.4 displays the weights of portfolio 4 (Hedge Funds HFR). On the whole, similarly to Portfolio 3, we observe that styles with low risk such as Convertible Arbitrage, Equity Market Neutral and Statistical Arbitrage tend to be included in the portfolios with a significant weight. Conversely, no or little weight is placed on hedge fund strategies with high risk (Emerging Markets (Total), Equity Non-Hedge, Fixed Income: Convertible Bonds). The only exceptions are Short Selling, because of its negative correlation with the other indices, and Equity Hedge, which has the highest mean return, therefore a high attractiveness.

| Global Minimum Risk Portfolios | | | | | |
|---|---|---|---|---|---|
| | Mean-Variance | Mean-VaR | Mean-adusted VaR | Mean-D.Dev. 0% | Omega 0% |
| **Portfolio 4 : Hedge Funds HFR** | | | | | |
| Convertible Arbitrage | 5% | 11% | 15% | 21% | 22% |
| Distressed Securities | - | - | - | - | - |
| Emerging Markets (Total) | - | - | - | - | - |
| Equity Hedge | 4% | 11% | 20% | 13% | 13% |
| Equity Market Neutral | 4% | 5% | 17% | 20% | 20% |
| Equity Non-Hedge | - | - | - | - | 4% |
| Event-Driven | - | - | - | - | - |
| Fixed Income: Arbitrage | 2% | - | - | - | - |
| Fixed Income: Convertible Bonds | - | - | - | - | - |
| Macro | 1% | - | 3% | - | 2% |
| Merger Arbitrage | 11% | 11% | - | 3% | 3% |
| Relative Value | 50% | 39% | - | - | - |
| Short Selling | 6% | 8% | 12% | 11% | 13% |
| Statistical Arbitrage | 17% | 15% | 34% | 32% | 22% |

Table 6.4: Global Minimum Risk Portfolios of HFR hedge fund style indices derived in different frameworks.

In Portfolio 4, the contrast between frameworks assuming normality and the other frameworks is even more important than for portfolio 3. The disparity is particularly obvious for indices such as Relative Value and Merger Arbitrage that exhibit a low risk in any setting. Though, they are almost exclusively included in the portfolios derived in the frameworks assuming normality. The reason is that these indices are among those exhibiting the highest kurtosis and negative skewness. Despite the fact that their risk ranking does not change much, their relative risk is nevertheless considerably increased with models taking into account moments of higher order than the variance. Conversely, indices showing zero or little positive skewness such as Equity Hedge, Equity Market Neutral, Short Selling and Statistical Arbitrage see their relative risk reduced with respect to highly negatively skewed indices. Therefore, they are underweighted in the frameworks assuming normality and gain weight in frameworks incorporating higher moments than variance.

The analysis conducted for both Portfolios 3 and 4 tends to confirm that frameworks assuming normality are not appropriate for hedge fund portfolio optimization. Looking at the statistics of the global minimum risk portfolios (see appendix 5, panel A), we observe that the models with no distributional assumption tend to increase the odd moments of the optimal portfolios and reduce all the even moments except the variance. Regarding the weights of Portfolios 3 and 4 in the frameworks that do not assume normality, we can observe that the HFR indices are distributed in a much more equal manner than the CSFB-Tremont indices. Nevertheless, the two portfolios tend to include the same strategy indices. In both portfolios, we find Equity Hedge (equivalent to Long-Short Equity in the CSFB-Tremont terminology), Equity Market Neutral and Short Selling (Dedicated Short Bias for CSFB-Tremont). More puzzling is the fact that despite similar statistical characteristics, Convertible Arbitrage has an important weight in Portfolio 4 (HFR) but is not included in Portfolio 3 (CSFB-Tremont); in the same way, Fixed Income Arbitrage is given no weight in Portfolio 4 (HFR) but is invested in Portfolio 3 (CSFB-Tremont).

### 6.2.3 Global Hedge Fund Style and Asset Classes Universe

In this analysis, we add asset class indices to the portfolios of hedge fund style indices. In general, we observe that all the settings incorporate WGBI. Conversely, MSCI is almost never attributed a significant share.

Appendix 5 (panel B) exhibits the basic statistics of the global minimum risk portfolios for Portfolios 5 and 6. Table 6.5 shows the weights for Portfolio 5 (Global CSFB). We see that WGBI is included in the portfolios (between 5% and 8%), mostly at the expense of Equity Market Neutral.

| Global Minimum Risk Portfolios | | | | | |
|---|---|---|---|---|---|
| | Mean-Variance | Mean-VaR | Mean-adusted VaR | Mean-D.Dev. 0% | Omega 0% |
| **Portfolio 5 : Global CSFB** | | | | | |
| Convertible Arbitrage | - | - | - | - | - |
| Dedicated Short Bias | 10% | 8% | 11% | 13% | 11% |
| Emerging Markets | - | - | 3% | 3% | 2% |
| Equity Market Neutral | 47% | 54% | 52% | 52% | 52% |
| Event Driven | 1% | 3% | - | - | - |
| Fixed Income Arbitrage | 27% | 23% | 14% | 13% | 14% |
| Global Macro | - | - | - | - | - |
| Long-Short Equity | 5% | 5% | 10% | 14% | 13% |
| Managed Futures | - | - | - | - | 2% |
| SSB WGBI | 8% | 6% | 8% | 5% | 5% |
| MSCI | 3% | 0% | 1% | 0% | 1% |

Table 6.5: Global Minimum Risk Portfolios of asset classes with CSFB-Tremont hedge fund style indices.

Table 6.6 shows the weights for Portfolio 6 (Global HFR). In this portfolio, WGBI tends to be included at the expense of Convertible Arbitrage and Equity Market Neutral. No portfolio invests in Fund of Funds. This is not a surprise since it is largely overlapping with the strategy indices. Only the mean-variance and the mean-VaR portfolios invest heavily in the composite Fixed Income (Total) index, because of its negative skewness and high kurtosis.

| Global Minimum Risk Portfolios | | | | | |
|---|---|---|---|---|---|
| | Mean-Variance | Mean-VaR | Mean-adusted VaR | Mean-D.Dev. 0% | Omega 0% |
| **Portfolio 6 : Global HFR** | | | | | |
| Convertible Arbitrage | 11% | 13% | 14% | 26% | 14% |
| Distressed Securities | - | - | - | - | 0% |
| Emerging Markets (Total) | - | - | - | - | - |
| Equity Hedge | 3% | 10% | 20% | 14% | 20% |
| Equity Market Neutral | 16% | 14% | 11% | 17% | 12% |
| Equity Non-Hedge | - | - | - | - | 2% |
| Event-Driven | - | - | - | - | 0% |
| Fixed Income (Total) | 22% | 20% | 5% | - | - |
| Fixed Income: Arbitrage | 7% | 2% | 4% | - | 3% |
| Fixed Income: Convertible Bonds | - | - | - | 0% | 2% |
| Fund of Funds | - | - | - | - | 0% |
| Macro | - | - | - | - | 3% |
| Merger Arbitrage | 15% | 14% | - | - | 0% |
| Relative Value | 5% | 6% | - | - | - |
| Short Selling | 6% | 8% | 12% | 11% | 12% |
| Statistical Arbitrage | 9% | 8% | 22% | 26% | 19% |
| SSB WGBI | 8% | 6% | 12% | 6% | 12% |
| MSCI | - | - | - | - | 0% |

Table 6.6: Global Minimum Risk Portfolio of asset classes with HFR hedge fund style indices.

Overall, the observations we made in section 6.2.2 for Portfolios 3 and 4 are still relevant here. In addition, if we compare Portfolios 5 and 6 on one side and Portfolios 1 and 2 on the other side, we see that the global weighting attributed to the hedge fund class is much higher when strategy indices are available. However, the comparison is rather unfair since the inclusion of multiple hedge fund style indices allows a much better risk diversification.

### 6.2.4 Impact of Higher Loss Thresholds

Regarding portfolio optimization in the mean-downside deviation and omega settings, we can expect the use of different target returns or loss thresholds to have an impact on the results obtained. Indeed, in the omega framework, a rise of the loss threshold increases the risk perception of an asset and reduces the reward perception. This phenomenon can be observed in the upper graph of Figure 6.1 that represents the minimum risk frontiers derived in the omega setting for Portfolio 6 at loss thresholds of 0%, 0.1% and 0.63%. As a consequence of the definition of $I_1(r)$ and $I_2(r)$, the frontier shifts to the lower right of the graph. The lower graph of Figure 6.1 shows the minimum risk portfolios in the mean-downside deviation framework for the same return targets. There, the frontier shifts to the right only because the reward (the expected return) is estimated on the entire distribution and is thus not affected.

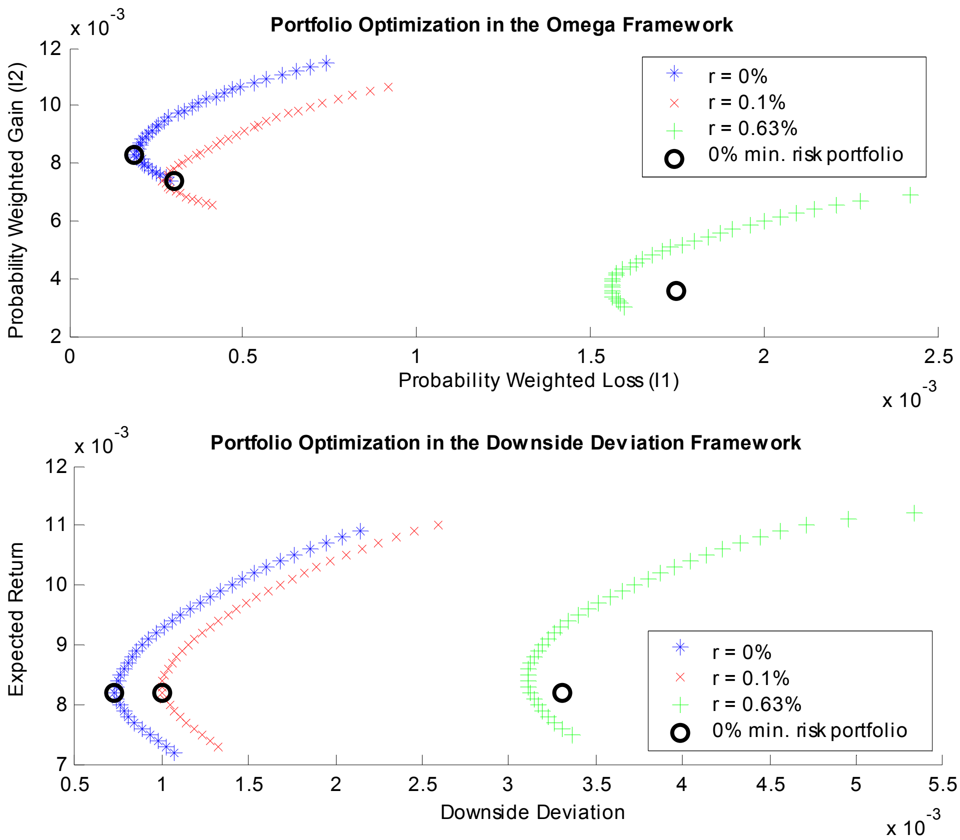


Figure 6.1: Optimal portfolios in the omega and mean-downside deviation frameworks for Portfolio 6 at three different levels of return. Each framework presents the sets of optimal portfolios in its own risk/reward dimensions. The black circles indicate the position of the global minimum risk portfolio derived at a zero return level and reevaluated at higher return levels. It can be noted that for a threshold of 0%, omega and mean-downside deviation tend to be represented on the same scale for their reward. From the mathematical definition of omega we can show that $I_2(0) = \mu + I_1(0)$. As $I_1(0)$ tends to be very low in a well diversified portfolio, the relationship $I_2(0) \approx \mu$ holds.

The black circles represent the sensitivity of the global minimum risk portfolio derived at a zero return level to a shift in the threshold. At a 0.1% threshold, it appears that the portfolio tends to remain on the efficient set. However, for returns levels of 0.63%, it is particularly obvious that the portfolio is no longer efficient. For both frameworks, this can be explained by the fact that the weights of the optimal portfolios show little change between the thresholds of 0% and 0.1%, but the differences relative to the threshold of 0.63% are more important. It suggests that a small change of the loss threshold (10 basis points in the present case) may not necessarily lead to rebalancing the portfolio.

Table 6.7 displays the composition of the global minimum risk portfolios for Portfolio 6 in the mean-downside deviation and omega settings. In both settings, the weights of the portfolios at the threshold of 0.63% tend to move accordingly. In general, a relative change of the riskiness of an index leads to the change of its weight. This is the case for instance for equity market neutral, on which less weight is placed as its risk relatively to other indices increases with a high threshold. Equity Hedge however, sees its weight increased despite its augmented risk. This is because of its high reward properties that make it attractive for higher thresholds. Short Selling, which still exhibit the highest risk at the 0.63% threshold, confirms its important contribution to diversification (negative correlation) by keeping its weights almost unchanged. Finally, it is interesting to note the apparition of assets showing important negative skewness and high kurtosis that were not included in the portfolios before. This is the case of Fixed Income (Total) and Merger Arbitrage.

| Global Minimum Risk Portfolios | | | | | | |
|---|---|---|---|---|---|---|
| | Mean-D.Dev. 0% | Mean-D.Dev. 0.1% | Mean-D.Dev. 0.63% | Omega 0% | Omega 0.1% | Omega 0.63% |
| **Portfolio 6 : Global HFR** | | | | | | |
| Convertible Arbitrage | 26% | 25% | 17% | 14% | 15% | 14% |
| Distressed Securities | - | - | - | 0% | - | - |
| Emerging Markets (Total) | - | - | - | - | - | - |
| Equity Hedge | 14% | 17% | 25% | 20% | 19% | 28% |
| Equity Market Neutral | 17% | 14% | 9% | 12% | 19% | 8% |
| Equity Non-Hedge | - | - | - | 2% | 4% | - |
| Event-Driven | - | - | - | 0% | 0% | - |
| Fixed Income (Total) | - | - | 9% | - | 0% | 9% |
| Fixed Income: Arbitrage | - | - | 0% | 3% | 3% | 0% |
| Fixed Income: Convertible Bonds | 0% | - | - | 2% | 0% | - |
| Fund of Funds | - | - | - | 0% | - | - |
| Macro | - | - | - | 3% | 0% | - |
| Merger Arbitrage | - | - | - | 0% | 1% | 12% |
| Relative Value | - | - | - | - | 0% | 0% |
| Short Selling | 11% | 12% | 13% | 12% | 13% | 13% |
| Statistical Arbitrage | 26% | 26% | 18% | 19% | 16% | 9% |
| SSB WGBI | 6% | 6% | 9% | 12% | 10% | 7% |
| MSCI | - | - | - | 0% | 0% | - |

Table 6.7: Global Minimum Risk Portfolios derived in the mean-downside deviation and omega settings for different loss thresholds.

Appendices 6a and 6b provide the statistics of the global minimum risk portfolios derived in the mean-downside deviation and omega frameworks with the different thresholds. As it can be observed, the mean returns of the portfolios tend to increase with higher thresholds. This is due to the fact that indices with too low expected returns tend to be excluded. In general, all the other moments show no or little change.

### 6.2.5 Impact of Time Varying Loss Thresholds

Portfolio optimization in the omega framework with a synchronized threshold may also result in significantly different results comparatively to optimization with a fixed threshold. We can expect the discrepancies to be particularly important when the benchmark time series exhibits high volatility. Figure 6.2 shows the optimal sets derived under the different thresholds for Portfolio 5 (Global CSFB). With a synchronized Libor threshold, the set of optimal portfolios is considered as more risky because the 0.1% value used for the fixed thresholds (Libor rate as of July 2002) is among the lowest of the whole time series. Therefore, the synchronized threshold is on average higher than the fixed threshold. The converse can be observed for the Fund of Funds synchronized threshold, for which optimal portfolios are less risky than those derived with a fixed threshold of 0.63% (mean return of Fund of Funds).

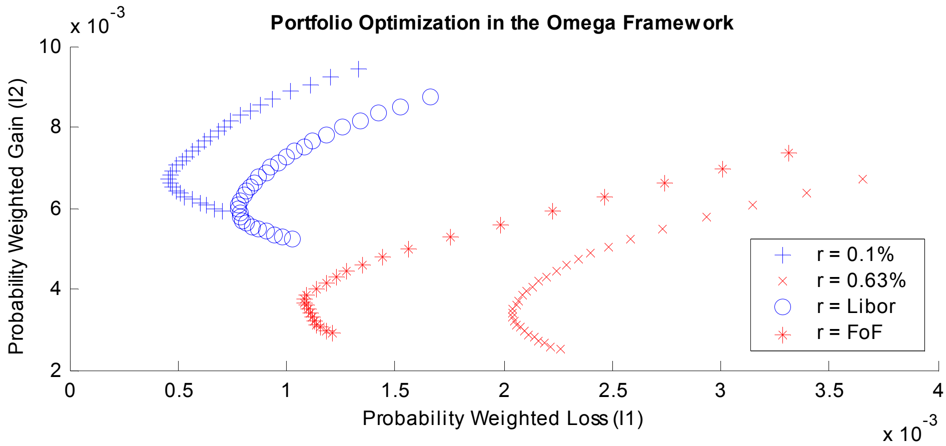


Figure 6.2: Efficient Frontiers derived in the omega framework for Portfolio 5 with fixed thresholds and synchronized thresholds.

Table 6.8 presents the composition of the global minimum risk portfolios for Portfolio 5 (Global CSFB) in the omega setting for the fixed and synchronized thresholds.

| Global Minimum Risk Portfolios | | | | |
|---|---|---|---|---|
| | Omega 0.1% | Omega 0.63% | Omega XS Libor | Omega XS FoF |
| **Portfolio 5 : Global CSFB** | | | | |
| Convertible Arbitrage | - | - | - | 16% |
| Dedicated Short Bias | 11% | 7% | 10% | 1% |
| Emerging Markets | 2% | - | 1% | 8% |
| Equity Market Neutral | 54% | 57% | 56% | 22% |
| Event Driven | 1% | 7% | 3% | 13% |
| Fixed Income Arbitrage | 15% | 16% | 16% | 0% |
| Global Macro | 0% | - | - | 14% |
| Long-Short Equity | 9% | 6% | 6% | 27% |
| Managed Futures | 1% | 2% | 0% | - |
| SSB WGBI | 7% | 6% | 8% | - |
| MSCI | 1% | - | 0% | - |

Table 6.8: Global Minimum Risk Portfolios derived in the omega framework for fixed thresholds and synchronized thresholds.

There is no important change in the composition of portfolios between the fixed 0.1% threshold and the moving Libor threshold. Indeed, the Libor has a very low variation over time (volatility of 0.08%). On the contrary, the comparison of the fixed 0.63% threshold with the Fund of Funds synchronized threshold exhibits significant changes. Not surprisingly, we observe as a rule that the indices showing an improved relative position in the risk ordering see their weight increased. The converse is true for indices which position is deteriorated in the risk ordering (see appendix 4 for the risk ordering of the indices).

It is interesting to note that Portfolio 5 exhibits more equally distributed weights under a Fund of Funds synchronized threshold than with the fixed 0.63% threshold. However, we should not consider this as a generalization because Portfolio 6 (not shown here for brevity reasons) exhibits the opposite trend.

### 6.2.6 Efficiency of Omega Portfolio Optimization

As we argue in section 5.1 that omega provides a more appropriate measure of risk and reward, asset allocation in this framework should determine a frontier at least as efficient as those derived in other settings. In other words, for the same risk exposure, omega optimization should provide a better or equivalent reward. Figure 6.3 compares the omega efficient frontier for Portfolio 5 (Global CSFB) at a return level of 0% with those derived in other settings. In order to display the frontiers in comparable spaces, we compute the values of $I_1(0)$ and $I_2(0)$ for the optimal portfolios obtained in the mean-variance, mean-VaR, mean-adjusted VaR and mean-downside deviation frameworks.

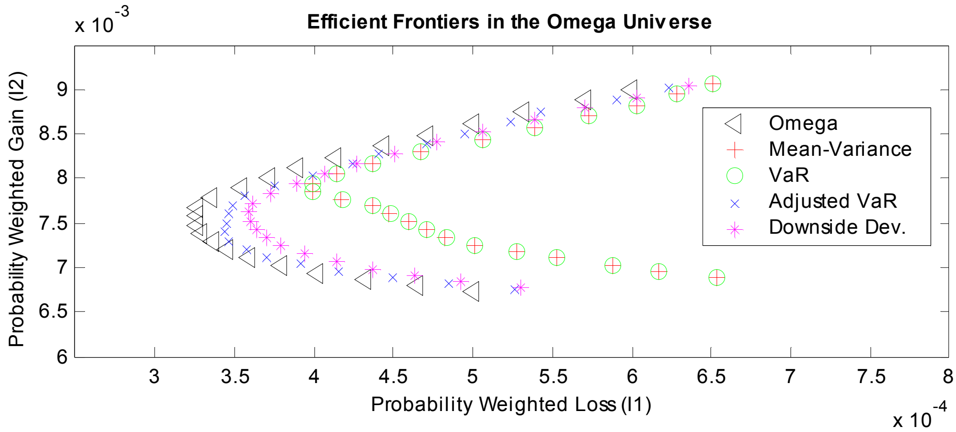


Figure 6.3: Comparison of Efficient Frontiers in the Omega Universe for Portfolio 5 (Global CSFB)

We observe that the omega frontier dominates the other opportunity sets. Because the optimizations are performed with no short-sell constraints, the frontiers converge toward each end. However, the efficient sets of models assuming normality tends to be quite different from those derived in the frameworks without this assumption. In particular, the minimum risk portfolios frontier is not symmetric for mean-variance and mean-VaR[7]. In theory, if the returns were normally distributed, the frontiers would coincide. This is indeed the case for

[7] Note that for frameworks other than the one used as universe of comparison, the global minimum risk portfolio may no longer be the one located at the extreme left of the frontier. This is in general the case for the mean-variance optimal set, where the minimum variance portfolio lies lower, on the inefficient part of the frontier.

Portfolio 1 (Global HFI) which is normally distributed and Portfolio 2 (Global FoF) that is not far from a normal distribution[8].

In order to compare the efficiency of the different frameworks with reference to a traditional universe such as mean-variance, we also represent in Figure 6.4 the optimal sets in the standard deviation / expected return spaces. Logically, the mean-variance optimization exhibits the most efficient set, since it provides the only appropriate measures of risk and reward in this universe, and the same conclusions as above can be drawn but in favor of mean-variance analysis. Nevertheless we can see in this universe that omega produces an optimal set that is not very different from those of mean-adjusted VaR and mean-downside deviation.

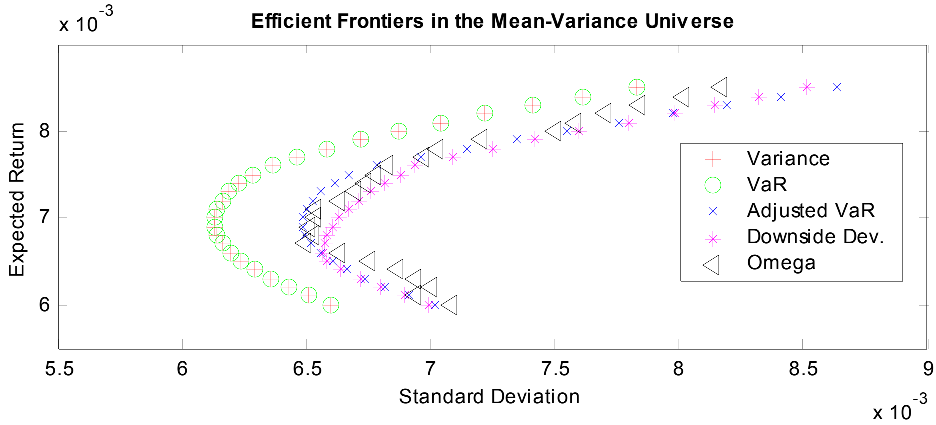


Figure 6.4: Comparison of Efficient Frontiers in the Mean-Variance universe for Portfolio 5 (global CSFB)

By focusing on the global minimum risk portfolios in the omega universe, we find that the risk of the omega portfolio represents 67.4% of the risk of the mean-variance portfolio (see table 6.9). Alternatively, in the mean-variance universe, the risk of the mean-variance portfolio is only 91% of that of omega. The analysis of Portfolios 3, 4 and 6 leads to the same conclusions.

**Global Minimum Risk Portfolios : Risk Diversification Efficiency**

| | Optimization Method | | | | |
|---|---|---|---|---|---|
| Universe of Reference | Mean-Variance | Mean-VaR | Mean-adusted VaR | Mean-D.Dev. 0% | Omega 0% |
| Mean-Variance | 91.0% | 92.4% | 97.3% | 101.1% | 100.0% |
| Omega 0% | 100.0% | 90.6% | 71.6% | 74.3% | 67.4% |

Table 6.9: Risk Diversification Efficiency for Portfolio 5 in two universes, mean-variance and omega. In the mean-variance universe, the risk of the mean-variance global minimum risk portfolio represents 91% of that of the omega global minimum risk portfolio taken as the reference. In the omega universe, the risk of the omega portfolio is 67.4% of that of the mean-variance portfolio, taken as the reference in this case.

[8] Showing these graphs is of little interest, as all the frontiers are overlapping.

A way to assess the efficiency of omega with respect to other settings is to compare the attractiveness of the global minimum risk portfolios. Table 6.10 shows below the expected returns and omegas at a zero return level achieved by the global minimum risk portfolios in the different settings (Portfolios 3 to 6). We find that the omega optimization never provides a lower expected return than the other settings. For a specific Portfolio, other frameworks may though display the same expected return as the omega setting, but no one is able to do it for each Portfolio. By observing the omega values in the different frameworks, we observe that mean-adjusted VaR, mean-downside deviation and omega tend to form a homogeneous group, clearly distinct from the two frameworks assuming normality. But of course, the omega setting always displays the highest omega value.

| Global Minimum Risk Portfolios | | | | | |
|---|---|---|---|---|---|
| Statistics | Mean-Variance | Mean-VaR | Mean-adusted VaR | Mean-D.Dev. 0% | Omega 0% |
| **Portfolio 3 : Hedge Funds CSFB** | | | | | |
| Mean return | 0.74% | 0.76% | 0.76% | 0.76% | 0.76% |
| Omega 0% | 14.04 | 16.53 | 20.60 | 19.50 | 21.60 |
| **Portfolio 4 : Hedge Funds HFR** | | | | | |
| Mean return | 0.77% | 0.82% | 0.84% | 0.82% | 0.84% |
| Omega 0% | 15.37 | 17.93 | 30.40 | 31.12 | 33.30 |
| **Portfolio 5 : Global CSFB** | | | | | |
| Mean return | 0.70% | 0.74% | 0.73% | 0.74% | 0.74% |
| Omega 0% | 15.20 | 17.58 | 21.72 | 21.25 | 23.28 |
| **Portfolio 6 : Global HFR** | | | | | |
| Mean return | 0.75% | 0.80% | 0.81% | 0.82% | 0.83% |
| Omega 0% | 18.66 | 21.76 | 38.22 | 40.73 | 43.98 |

Table 6.10: Expected Returns and Omegas for the global minimum risk portfolios in the different settings.

The general conclusions we can draw from all these examples are that risk measures considering the downside tend to be more efficient than those focusing on both the downside and the upside. However, relying on a limited number of moments to take into account the asymmetry and the fat tail properties of the distribution may not be sufficient to fully benefit from the risk diversification and performance enhancement effects of assets with non-normally distributed returns. The omega measure provides a better definition of risk and reward and may therefore outperform optimization carried out with less accurate measures.

# 7 Conclusion

As we have seen, hedge fund returns can be seriously biased, which makes the traditional mean-variance analysis inappropriate. In this paper, we emphasize the importance of including higher moments in the investment decision problem. It has been shown that investors care about all the moments of the returns distribution and this should be reflected in portfolio evaluation methods. We show that alternative models including a limited number of moments in the analysis are in general insufficient to adequately assess investment opportunities. In addition, these measures may suffer from the fact that a characterization of the form of the utility function is required.

Omega is a function that takes into account all the moments of the returns distribution. Thus, it can be accommodated with any asset showing non-normally distributed returns, such as hedge funds, bonds or equity in illiquid markets. For the comparison of investment opportunities, the measure requires no assumption on the utility function. Moreover, even if the returns are normally distributed, omega can provide additional information to mean-variance analysis by incorporating the investor's perception of loss and gain. Finally, omega presents a more accurate picture of the statistical properties of historical returns since it is directly computed from the distribution itself.

In this paper we carry out hedge fund performance ranking and portfolio optimization with different methods. Not surprisingly, we find that frameworks incorporating higher moments than the variance provide more consistent results than those assuming normality. In particular, they correct for the negative skewness and excess kurtosis properties of hedge fund returns. However, we show that higher moments matter; as omega is the only measure embodying all the moments of the returns distribution, it produces markedly different results. Given that omega offers the most accurate definition of risk and reward, our results suggest that omega optimization provides enhanced capabilities for risk diversification and reward enhancement when returns are not normally distributed. If returns are normally distributed, omega may still contribute to the investment analysis by considering the specific threshold below which an investor considers a given return as a loss.

The flexibility introduced in the analysis by the investor's specific return level is a characteristic shared by both omega and the downside deviation measures. The downside deviation is a lower partial moment measure of order 2 implying that the investor's preferences are limited to skewness. Thus, the mean-downside deviation framework fails to fully capture the investor's preferences. We find that for identical return levels, omega optimization can provide markedly different results.

In this paper, we made several simplifications or assumptions. First, our analysis implies that historical returns will be replicated in the future. To appropriately test for the robustness of our results, we should analyze the out-of-sample performance of the optimal portfolios. But the problem of scarcity of data in the hedge fund industry can seriously bias the results of such an analysis. Indeed, at most 120 data points are available, which is insufficient to reliably perform in-sample and out-of-sample analyses. Second, we assume that the return level of the omega function is exogenously defined, which does not exactly reflect the reality since the loss threshold is defined by the investor's preferences. Finally, it is worth to note that we do not specifically account for serial correlation, a typical hedge fund data property. Similarly, the data may not be totally free of the survivorship bias.

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## Appendix 1

| Historical Data Statistical Properties | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Mean | Minimum | Maximum | Standard Dev. | Skewness | Kurtosis | 5th moment [1] | 6th moment [1] | 7th moment [1] | 8th moment [1] | JB Statistic [2] |
| **CSFB-Tremont Indices** | | | | | | | | | | | |
| Hedge Fund Index | 0.89% | -7.55% | 8.53% | 2.63% | 0.1 | 4.1 | -0.3 | 25.9 | -11.9 | 204.5 | 4.3 |
| Convertible Arbitrage | 0.82% | -4.68% | 3.57% | 1.41% | -1.6 | 6.9 | -21.3 | 81.8 | -294.4 | 1121.0 | **103.2** |
| Dedicated Short Bias | 0.13% | -8.69% | 22.71% | 5.31% | 0.9 | 5.0 | 15.2 | 64.9 | 260.4 | 1101.0 | **28.7** |
| Emerging Markets | 0.55% | -23.03% | 16.42% | 5.48% | -0.5 | 5.8 | -11.7 | 76.4 | -249.8 | 1281.2 | **34.4** |
| Equity Market Neutral | 0.89% | -1.15% | 3.26% | 0.93% | 0.1 | 2.9 | 1.2 | 12.5 | 10.1 | 61.3 | 0.2 |
| Event Driven | 0.86% | -11.78% | 3.68% | 1.84% | -3.3 | 22.9 | -151.5 | 1040.1 | -7147.1 | 49209.0 | **1797.5** |
| Fixed Income Arbitrage | 0.59% | -6.96% | 2.02% | 1.16% | -3.5 | 20.6 | -125.2 | 794.8 | -5114.7 | 33181.0 | **1472.9** |
| Global Macro | 1.18% | -11.55% | 10.60% | 3.76% | 0.0 | 4.3 | -1.8 | 29.0 | -33.9 | 248.5 | 5.9 |
| Long-Short Equity | 1.00% | -11.44% | 13.01% | 3.40% | 0.2 | 5.5 | 1.0 | 55.1 | -1.2 | 651.7 | **24.8** |
| Managed Futures | 0.54% | -9.35% | 9.95% | 3.43% | 0.1 | 4.0 | 0.0 | 23.2 | -3.8 | 155.8 | 3.4 |
| **HFR Indices** | | | | | | | | | | | |
| Convertible Arbitrage | 0.88% | -3.19% | 3.33% | 1.04% | -1.3 | 6.0 | -16.6 | 65.5 | -222.8 | 851.0 | **61.6** |
| Distressed Securities | 0.82% | -8.50% | 5.06% | 1.66% | -1.8 | 11.8 | -56.4 | 326.0 | -1796.2 | 10194.0 | **363.4** |
| Emerging Markets (Total) | 0.58% | -21.02% | 14.80% | 4.68% | -0.7 | 6.6 | -18.3 | 107.2 | -422.4 | 2147.0 | **60.0** |
| Equity Hedge | 1.26% | -7.65% | 10.88% | 2.89% | 0.3 | 4.3 | 3.4 | 33.3 | 40.8 | 317.5 | **7.0** |
| Equity Market Neutral | 0.78% | -1.67% | 3.59% | 0.98% | 0.0 | 3.2 | 0.5 | 15.6 | 6.6 | 93.6 | 0.1 |
| Equity Non-Hedge | 1.08% | -13.34% | 10.74% | 4.40% | -0.4 | 3.3 | -4.4 | 19.9 | -44.3 | 164.2 | 3.6 |
| Event-Driven | 1.03% | -8.90% | 5.13% | 2.01% | -1.3 | 8.0 | -31.2 | 154.7 | -731.4 | 3598.0 | **131.7** |
| Fixed Income (Total) | 0.70% | -3.27% | 3.28% | 0.97% | -1.2 | 7.0 | -21.0 | 94.3 | -346.9 | 1463.7 | **88.3** |
| Fixed Income: Arbitrage | 0.48% | -6.45% | 3.04% | 1.29% | -2.8 | 15.7 | -79.0 | 419.3 | -2206.4 | 11679.0 | **793.2** |
| Fixed Income: Convertible Bonds | 0.70% | -13.06% | 14.42% | 4.11% | -0.5 | 5.3 | -4.5 | 44.7 | -38.4 | 437.4 | **24.1** |
| Fund of Funds | 0.63% | -7.47% | 6.85% | 1.89% | -0.2 | 6.2 | -9.1 | 80.9 | -215.1 | 1308.9 | **42.1** |
| Macro | 0.83% | -6.40% | 6.82% | 2.27% | 0.0 | 3.6 | -1.3 | 20.9 | -21.1 | 157.0 | 1.2 |
| Merger Arbitrage | 0.91% | -5.69% | 2.47% | 1.13% | -2.5 | 13.5 | -71.1 | 401.9 | -2303.2 | 13337.0 | **557.7** |
| Relative Value | 0.75% | -2.96% | 2.01% | 0.74% | -1.9 | 9.5 | -40.0 | 188.5 | -890.4 | 4303.1 | **230.1** |
| Short Selling | 0.59% | -21.21% | 22.84% | 7.08% | 0.2 | 3.9 | 1.4 | 26.3 | 12.7 | 218.7 | 3.3 |
| Statistical Arbitrage | 0.70% | -2.00% | 3.60% | 1.14% | -0.2 | 3.0 | -0.9 | 13.2 | -3.4 | 68.7 | 0.7 |
| **Traditional Indices** | | | | | | | | | | | |
| SSB WGBI | 0.57% | -5.08% | 7.71% | 2.21% | 0.2 | 3.7 | 3.2 | 24.0 | 39.7 | 199.0 | 2.1 |
| MSCI | 0.40% | -13.45% | 8.91% | 4.13% | -0.6 | 3.3 | -5.4 | 21.2 | -54.4 | 190.1 | 5.5 |

[1] centralized normalized moments

[2] in bold : normality is rejected at a 5% level

# Appendix 2

## Risk, Reward and Performance measures for asset allocation frameworks

| Framework | Risk | Reward | Performance |
|---|---|---|---|
| Mean – Variance | $\sigma_R = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\mu_R - R_i)^2}$ | $\mu_R = \frac{1}{n}\sum_{i=1}^{n} R_i$ | $Sharpe\ ratio = \frac{\mu_R - R_f}{\sigma_R}$ |
| Mean – Value-at-Risk | $VaR_{99\%} = \lvert \mu_R - 2.33 \times \sigma_R \rvert$ | $\mu_R$ | $Modified\ Sharpe = \frac{\mu_R - R_f}{R_f - VaR_{99\%}}$ |
| Mean – Adjusted Value-at-Risk | $adjusted\ VaR_{99\%} = \lvert \mu_R - z_{CF} \times \sigma_R \rvert$ | $\mu_R$ | $Adj. VaR\ Modif.\ Sharpe = \frac{\mu_R - R_f}{R_f - VaR_{99\%}}$ |
| Mean – Downside Deviation | $downside\ \ dev. = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\tau - R_i)^2\ \mathbf{1}_{R_i<\tau}}$ | $\mu_R$ | $Sortino\ ratio = \frac{\mu_R - R_f}{downside\ deviation}$ |
| Omega | $I_1(r) = \int_a^r F(x)\, dx$ | $I_2(r) = \int_r^b (1 - F(x))\, dx$ | $\Omega(r) = \frac{I_2(r)}{I_1(r)}$ |

$\mu_R$ is the expected return of the returns distribution.

$\sigma_R$ is the standard deviation of the returns distribution.

$R_i$ are the assets returns.

**1** is the indicator function.

$n$ is the sample size.

$R_f$ is the risk-free rate.

$z_c$ is the critical value of the normal standard distribution at a $(1-\alpha)$ threshold.

$z_{CF}$ is the adjustment of the critical value of the normal standard distribution for skewness and kurtosis by using the Cornish-Fisher (1937) expansion: $z_{CF} = z_c + \frac{1}{6}(z_c^2 - 1)S + \frac{1}{24}(z_c^3 - 3z_c)K - \frac{1}{36}(2z_c^3 - 5z_c)S^2$

where $S$ is the skewness of the return distribution,

$K$ is the excess kurtosis,

$\tau$ is the return level.

$F$ is the cumulative distribution function of the asset returns defined on the interval *[a, b]*.

$r$ is the loss threshold

# Appendix 3

**Performance Ranking : Panel A**

| | Sharpe | | VaR modified Sharpe | | Adj.VaR modified Sharpe | | Sortino | | | | | | Omega | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | 0.00% | | 0.10% | | 0.63% | | 0.00% | | 0.10% | | 0.63% | | Synchr. Libor | | Synchr. FoF | |
| CSFB-Tremont Indices | | | | | | | | | | | | | | | | | | | | | | |
| Hedge Fund Index | 0.30 | **5** | 0.15 | **5** | 0.14 | **3** | 0.64 | **5** | 0.55 | **4** | 0.16 | **5** | 2.32 | **5** | 2.10 | **5** | 1.22 | **6** | 1.88 | **5** | 1.69 | **1** |
| Convertible Arbitrage | 0.51 | **2** | 0.29 | **2** | 0.18 | **2** | 0.92 | **2** | 0.78 | **2** | 0.17 | **4** | 3.68 | **2** | 3.20 | **2** | 1.33 | **3** | 2.68 | **2** | 1.21 | **6** |
| Dedicated Short Bias | 0.01 | **12** | 0.00 | **12** | 0.00 | **12** | 0.04 | **12** | 0.01 | **12** | -0.13 | **12** | 0.99 | **12** | 0.94 | **12** | 0.72 | **12** | 0.89 | **12** | 0.75 | **12** |
| Emerging Markets | 0.08 | **10** | 0.04 | **10** | 0.03 | **11** | 0.15 | **10** | 0.12 | **10** | -0.02 | **7** | 1.19 | **11** | 1.13 | **10** | 0.88 | **7** | 1.08 | **10** | 0.87 | **8** |
| Equity Market Neutral | 0.84 | **1** | 0.61 | **1** | 0.66 | **1** | 3.45 | **1** | 2.70 | **1** | 0.50 | **1** | 10.62 | **1** | 8.05 | **1** | 1.90 | **1** | 5.70 | **1** | 1.38 | **5** |
| Event Driven | 0.41 | **4** | 0.22 | **4** | 0.08 | **8** | 0.66 | **4** | 0.57 | **3** | 0.16 | **6** | 3.15 | **4** | 2.77 | **3** | 1.28 | **4** | 2.38 | **3** | 1.57 | **2** |
| Fixed Income Arbitrage | 0.42 | **3** | 0.23 | **3** | 0.11 | **6** | 0.68 | **3** | 0.55 | **5** | -0.04 | **10** | 3.22 | **3** | 2.69 | **4** | 0.79 | **11** | 2.18 | **4** | 0.85 | **10** |
| Global Macro | 0.29 | **6** | 0.14 | **6** | 0.12 | **4** | 0.57 | **6** | 0.51 | **6** | 0.23 | **2** | 2.13 | **6** | 1.99 | **6** | 1.38 | **2** | 1.85 | **6** | 1.52 | **4** |
| Long-Short Equity | 0.27 | **7** | 0.13 | **7** | 0.11 | **5** | 0.55 | **7** | 0.48 | **7** | 0.18 | **3** | 2.04 | **7** | 1.88 | **7** | 1.23 | **5** | 1.73 | **7** | 1.54 | **3** |
| Managed Futures | 0.13 | **9** | 0.06 | **9** | 0.06 | **9** | 0.26 | **9** | 0.21 | **9** | -0.04 | **8** | 1.43 | **9** | 1.32 | **9** | 0.87 | **8** | 1.21 | **9** | 0.86 | **9** |
| SSB WGBI | 0.21 | **8** | 0.10 | **8** | 0.10 | **7** | 0.46 | **8** | 0.36 | **8** | -0.04 | **9** | 1.82 | **8** | 1.62 | **8** | 0.87 | **9** | 1.43 | **8** | 0.88 | **7** |
| MSCI | 0.07 | **11** | 0.03 | **11** | 0.03 | **10** | 0.14 | **11** | 0.10 | **11** | -0.07 | **11** | 1.20 | **10** | 1.13 | **11** | 0.82 | **10** | 1.06 | **11** | 0.78 | **11** |

**Performance Ranking : Panel B**

| | Sharpe | | VaR modified Sharpe | | Adj.VaR modified Sharpe | | Sortino | | | | | | Omega | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | 0.00% | | 0.10% | | 0.63% | | 0.00% | | 0.10% | | 0.63% | | Synchr. Libor | | Synchr. FoF | |
| HFR Indices | | | | | | | | | | | | | | | | | | | | | | |
| Convertible Arbitrage | 0.75 | **2** | 0.50 | **2** | 0.30 | **3** | 1.65 | **3** | 1.39 | **3** | 0.34 | **2** | 6.41 | **3** | 5.38 | **3** | 1.74 | **2** | 4.24 | **3** | 1.42 | **4** |
| Distressed Securities | 0.43 | **8** | 0.24 | **8** | 0.11 | **10** | 0.80 | **9** | 0.68 | **9** | 0.15 | **7** | 3.29 | **8** | 2.84 | **8** | 1.23 | **7** | 2.40 | **9** | 1.38 | **5** |
| Emerging Markets (Total) | 0.10 | **16** | 0.05 | **16** | 0.03 | **16** | 0.18 | **16** | 0.15 | **16** | -0.01 | **15** | 1.25 | **16** | 1.18 | **16** | 0.88 | **15** | 1.12 | **16** | 0.89 | **14** |
| Equity Hedge | 0.40 | **9** | 0.21 | **9** | 0.20 | **7** | 0.93 | **7** | 0.83 | **7** | 0.38 | **1** | 2.92 | **9** | 2.67 | **9** | 1.64 | **3** | 2.41 | **8** | 2.46 | **1** |
| Equity Market Neutral | 0.69 | **4** | 0.45 | **4** | 0.45 | **1** | 2.26 | **1** | 1.80 | **1** | 0.25 | **5** | 7.22 | **2** | 5.68 | **2** | 1.41 | **5** | 4.35 | **2** | 1.16 | **8** |
| Equity Non-Hedge | 0.22 | **13** | 0.11 | **13** | 0.09 | **13** | 0.39 | **14** | 0.35 | **14** | 0.15 | **8** | 1.71 | **14** | 1.62 | **13** | 1.21 | **8** | 1.53 | **13** | 1.34 | **7** |
| Event-Driven | 0.46 | **7** | 0.26 | **7** | 0.14 | **9** | 0.87 | **8** | 0.77 | **8** | 0.29 | **4** | 3.37 | **7** | 2.99 | **7** | 1.55 | **4** | 2.63 | **7** | 2.10 | **2** |
| Fixed Income (Total) | 0.62 | **5** | 0.38 | **5** | 0.22 | **5** | 1.35 | **5** | 1.10 | **6** | 0.10 | **10** | 5.42 | **5** | 4.32 | **5** | 1.11 | **10** | 3.39 | **5** | 1.05 | **10** |
| Fixed Income: Arbitrage | 0.29 | **11** | 0.15 | **11** | 0.07 | **14** | 0.50 | **12** | 0.38 | **12** | -0.13 | **18** | 2.47 | **10** | 2.05 | **11** | 0.60 | **18** | 1.67 | **11** | 0.74 | **17** |
| Fixed Income: Convertible Bonds | 0.15 | **15** | 0.07 | **15** | 0.05 | **15** | 0.25 | **15** | 0.21 | **15** | 0.02 | **12** | 1.46 | **15** | 1.36 | **15** | 0.96 | **12** | 1.27 | **15** | 0.95 | **12** |
| Fund of Funds | 0.28 | **12** | 0.14 | **12** | 0.10 | **12** | 0.60 | **11** | 0.48 | **11** | 0.00 | **13** | 2.21 | **12** | 1.92 | **12** | 0.90 | **14** | 1.65 | **12** | n.a | |
| Macro | 0.32 | **10** | 0.16 | **10** | 0.15 | **8** | 0.71 | **10** | 0.60 | **10** | 0.14 | **9** | 2.47 | **11** | 2.19 | **10** | 1.17 | **9** | 1.93 | **10** | 1.35 | **6** |
| Merger Arbitrage | 0.71 | **3** | 0.47 | **3** | 0.21 | **6** | 1.31 | **6** | 1.13 | **4** | 0.32 | **3** | 5.46 | **4** | 4.67 | **4** | 1.75 | **1** | 3.89 | **4** | 1.44 | **3** |
| Relative Value | 0.87 | **1** | 0.66 | **1** | 0.30 | **2** | 1.93 | **2** | 1.59 | **2** | 0.21 | **6** | 9.03 | **1** | 7.12 | **1** | 1.40 | **6** | 5.10 | **1** | 1.13 | **9** |
| Short Selling | 0.07 | **18** | 0.03 | **18** | 0.03 | **17** | 0.13 | **18** | 0.11 | **17** | -0.01 | **14** | 1.15 | **18** | 1.11 | **18** | 0.91 | **13** | 1.07 | **17** | 0.91 | **13** |
| Statistical Arbitrage | 0.53 | **6** | 0.31 | **6** | 0.29 | **4** | 1.41 | **4** | 1.12 | **5** | 0.09 | **11** | 4.31 | **6** | 3.46 | **6** | 1.10 | **11** | 2.74 | **6** | 1.01 | **11** |
| SSB WGBI | 0.21 | **14** | 0.10 | **14** | 0.10 | **11** | 0.46 | **13** | 0.36 | **13** | -0.04 | **16** | 1.82 | **13** | 1.62 | **14** | 0.87 | **16** | 1.43 | **14** | 0.88 | **15** |
| MSCI | 0.07 | **17** | 0.03 | **17** | 0.03 | **18** | 0.14 | **17** | 0.10 | **18** | -0.07 | **17** | 1.20 | **17** | 1.13 | **17** | 0.82 | **17** | 1.06 | **18** | 0.78 | **16** |

# Appendix 4

**Risk Ordering : Panel A**

| | Sigma | | VaR | | Adjusted VaR | | Downside deviation | | | | | | Omega : probability weighted loss ($l_1$) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | 0.00% | | 0.10% | | 0.63% | | 0.00% | | 0.10% | | 0.63% | | Synchr. Libor | | Synchr. FoF | |
| **CSFB-Tremont Indices** | | | | | | | | | | | | | | | | | | | | | | |
| Hedge Fund Index | 2.63% | **6** | 5.22% | **6** | 5.68% | **5** | 1.38% | **6** | 1.43% | **6** | 1.67% | **6** | 0.62% | **5** | 0.65% | **5** | 0.86% | **5** | 0.69% | **5** | 0.33% | **1** |
| Convertible Arbitrage | 1.41% | **3** | 2.47% | **3** | 4.07% | **2** | 0.88% | **3** | 0.92% | **3** | 1.10% | **3** | 0.29% | **3** | 0.31% | **3** | 0.45% | **3** | 0.34% | **3** | 0.58% | **4** |
| Dedicated Short Bias | 5.31% | **11** | 12.23% | **12** | 9.76% | **10** | 3.33% | **11** | 3.39% | **11** | 3.71% | **11** | 2.00% | **12** | 2.05% | **12** | 2.34% | **12** | 2.10% | **12** | 2.82% | **12** |
| Emerging Markets | 5.48% | **12** | 12.18% | **11** | 17.19% | **12** | 3.73% | **12** | 3.78% | **12** | 4.05% | **12** | 1.91% | **11** | 1.95% | **11** | 2.20% | **11** | 2.01% | **11** | 1.66% | **11** |
| Equity Market Neutral | 0.93% | **1** | 1.29% | **1** | 1.20% | **1** | 0.26% | **1** | 0.29% | **1** | 0.51% | **1** | 0.09% | **1** | 0.11% | **1** | 0.26% | **1** | 0.14% | **1** | 0.52% | **3** |
| Event Driven | 1.84% | **4** | 3.43% | **4** | 8.93% | **9** | 1.31% | **5** | 1.33% | **5** | 1.48% | **4** | 0.36% | **4** | 0.39% | **4** | 0.55% | **4** | 0.42% | **4** | 0.35% | **2** |
| Fixed Income Arbitrage | 1.16% | **2** | 2.11% | **2** | 4.60% | **3** | 0.86% | **2** | 0.89% | **2** | 1.04% | **2** | 0.24% | **2** | 0.26% | **2** | 0.40% | **2** | 0.29% | **2** | 0.66% | **6** |
| Global Macro | 3.76% | **9** | 7.56% | **9** | 8.73% | **8** | 2.09% | **9** | 2.13% | **8** | 2.37% | **8** | 0.95% | **8** | 0.99% | **8** | 1.18% | **8** | 1.02% | **8** | 0.89% | **7** |
| Long-Short Equity | 3.40% | **7** | 6.91% | **7** | 8.31% | **7** | 1.83% | **7** | 1.87% | **7** | 2.12% | **7** | 0.85% | **7** | 0.89% | **7** | 1.10% | **7** | 0.93% | **7** | 0.60% | **5** |
| Managed Futures | 3.43% | **8** | 7.44% | **8** | 8.01% | **6** | 2.09% | **8** | 2.14% | **9** | 2.41% | **9** | 1.04% | **9** | 1.09% | **9** | 1.37% | **9** | 1.14% | **9** | 1.54% | **10** |
| SSB WGBI | 2.21% | **5** | 4.58% | **5** | 4.61% | **4** | 1.24% | **4** | 1.29% | **4** | 1.57% | **5** | 0.62% | **6** | 0.66% | **6** | 0.91% | **6** | 0.70% | **6** | 1.19% | **8** |
| MSCI | 4.13% | **10** | 9.19% | **10** | 10.72% | **11** | 2.91% | **10** | 2.96% | **10** | 3.24% | **10** | 1.52% | **10** | 1.56% | **10** | 1.80% | **10** | 1.61% | **10** | 1.43% | **9** |

**Risk Ordering : Panel B**

| | Sigma | | VaR | | Adjusted VaR | | Downside deviation | | | | | | Omega : probability weighted loss ($l_1$) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | 0.00% | | 0.10% | | 0.63% | | 0.00% | | 0.10% | | 0.63% | | Synchr. Libor | | Synchr. FoF | |
| **HFR Indices** | | | | | | | | | | | | | | | | | | | | | | |
| Convertible Arbitrage | 1.04% | **4** | 1.55% | **3** | 2.62% | **4** | 0.53% | **5** | 0.56% | **5** | 0.73% | **3** | 0.16% | **4** | 0.17% | **4** | 0.30% | **2** | 0.20% | **4** | 0.48% | **6** |
| Distressed Securities | 1.66% | **8** | 3.03% | **8** | 6.65% | **13** | 1.02% | **8** | 1.05% | **8** | 1.24% | **8** | 0.33% | **8** | 0.35% | **8** | 0.54% | **8** | 0.39% | **8** | 0.41% | **3** |
| Emerging Markets (Total) | 4.68% | **17** | 10.32% | **17** | 15.81% | **17** | 3.22% | **17** | 3.26% | **17** | 3.52% | **17** | 1.60% | **17** | 1.65% | **17** | 1.89% | **17** | 1.70% | **17** | 1.35% | **15** |
| Equity Hedge | 2.89% | **13** | 5.45% | **13** | 5.70% | **11** | 1.36% | **13** | 1.40% | **13** | 1.64% | **13** | 0.61% | **12** | 0.64% | **12** | 0.85% | **12** | 0.68% | **12** | 0.40% | **2** |
| Equity Market Neutral | 0.98% | **3** | 1.51% | **2** | 1.52% | **1** | 0.35% | **1** | 0.38% | **1** | 0.60% | **2** | 0.12% | **2** | 0.14% | **2** | 0.32% | **3** | 0.16% | **2** | 0.64% | **9** |
| Equity Non-Hedge | 4.40% | **16** | 9.16% | **15** | 10.60% | **14** | 2.74% | **14** | 2.78% | **14** | 3.04% | **14** | 1.35% | **15** | 1.39% | **15** | 1.61% | **15** | 1.44% | **15** | 1.12% | **13** |
| Event-Driven | 2.01% | **10** | 3.65% | **9** | 6.64% | **12** | 1.19% | **11** | 1.22% | **11** | 1.41% | **10** | 0.41% | **9** | 0.43% | **9** | 0.61% | **9** | 0.47% | **9** | 0.33% | **1** |
| Fixed Income (Total) | 0.97% | **2** | 1.55% | **4** | 2.78% | **5** | 0.52% | **4** | 0.54% | **4** | 0.73% | **4** | 0.15% | **3** | 0.17% | **3** | 0.34% | **5** | 0.19% | **3** | 0.47% | **5** |
| Fixed Income: Arbitrage | 1.29% | **7** | 2.52% | **7** | 5.17% | **9** | 0.96% | **7** | 0.99% | **7** | 1.16% | **7** | 0.29% | **7** | 0.31% | **7** | 0.50% | **7** | 0.34% | **7** | 0.83% | **11** |
| Fixed Income: Convertible Bonds | 4.11% | **14** | 8.86% | **14** | 12.15% | **16** | 2.77% | **15** | 2.81% | **15** | 3.05% | **15** | 1.24% | **14** | 1.28% | **14** | 1.51% | **14** | 1.33% | **14** | 1.05% | **12** |
| Fund of Funds | 1.89% | **9** | 3.77% | **10** | 5.47% | **10** | 1.05% | **9** | 1.10% | **9** | 1.34% | **9** | 0.46% | **10** | 0.50% | **10** | 0.72% | **10** | 0.55% | **10** | n.a | |
| Macro | 2.27% | **12** | 4.46% | **11** | 4.72% | **8** | 1.16% | **10** | 1.21% | **10** | 1.46% | **11** | 0.52% | **11** | 0.56% | **11** | 0.80% | **11** | 0.61% | **11** | 0.47% | **4** |
| Merger Arbitrage | 1.13% | **5** | 1.73% | **5** | 3.93% | **6** | 0.69% | **6** | 0.72% | **6** | 0.87% | **6** | 0.19% | **5** | 0.21% | **5** | 0.32% | **4** | 0.23% | **5** | 0.50% | **7** |
| Relative Value | 0.74% | **1** | 0.98% | **1** | 2.14% | **3** | 0.39% | **2** | 0.41% | **2** | 0.56% | **1** | 0.09% | **1** | 0.10% | **1** | 0.23% | **1** | 0.13% | **1** | 0.50% | **8** |
| Short Selling | 7.08% | **18** | 15.89% | **18** | 16.55% | **18** | 4.55% | **18** | 4.60% | **18** | 4.89% | **18** | 2.49% | **18** | 2.54% | **18** | 2.81% | **18** | 2.58% | **18** | 3.35% | **17** |
| Statistical Arbitrage | 1.14% | **6** | 1.95% | **6** | 2.09% | **2** | 0.50% | **3** | 0.54% | **3** | 0.79% | **5** | 0.20% | **6** | 0.23% | **6** | 0.44% | **6** | 0.27% | **6** | 0.68% | **10** |
| SSB WGBI | 2.21% | **11** | 4.58% | **12** | 4.61% | **7** | 1.24% | **12** | 1.29% | **12** | 1.57% | **12** | 0.62% | **13** | 0.66% | **13** | 0.91% | **13** | 0.70% | **13** | 1.19% | **14** |
| MSCI | 4.13% | **15** | 9.19% | **16** | 10.72% | **15** | 2.91% | **16** | 2.96% | **16** | 3.24% | **16** | 1.52% | **16** | 1.56% | **16** | 1.80% | **16** | 1.61% | **16** | 1.43% | **16** |

# Appendix 5

## Panel A

| Global Minimum Risk Portfolios | | | | | |
|---|---|---|---|---|---|
| Statistics | Mean-Variance | Mean-VaR | Mean-adusted VaR | Mean-D.Dev. 0% | Omega 0% |
| **Portfolio 3 : Hedge Funds CSFB** | | | | | |
| Mean | 0.74% | 0.76% | 0.76% | 0.76% | 0.76% |
| Standard Dev. | 0.64% | 0.65% | 0.70% | 0.70% | 0.69% |
| Skewness | -0.5 | -0.2 | 0.3 | 0.2 | 0.2 |
| Kurtosis | 3.4 | 2.9 | 2.5 | 2.4 | 2.6 |
| 5th moment | -4.3 | -1.5 | 2.2 | 1.9 | 2.0 |
| 6th moment | 19.3 | 12.9 | 10.3 | 8.9 | 10.6 |
| 7th moment | -37.7 | -9.9 | 16.3 | 13.6 | 15.4 |
| 8th moment | 139.5 | 70.2 | 56.1 | 44.4 | 58.0 |
| Omega 0% | 14.04 | 16.53 | 20.60 | 19.50 | 21.60 |
| $I_1$ | 0.056% | 0.048% | 0.038% | 0.040% | 0.036% |
| $I_2$ | 0.779% | 0.792% | 0.782% | 0.785% | 0.780% |
| **Portfolio 4 : Hedge Funds HFR** | | | | | |
| Mean | 0.77% | 0.82% | 0.84% | 0.82% | 0.84% |
| Standard Dev. | 0.62% | 0.63% | 0.72% | 0.71% | 0.69% |
| Skewness | -0.8 | -0.6 | 0.6 | 0.7 | 0.6 |
| Kurtosis | 6.2 | 5.5 | 3.9 | 4.6 | 4.2 |
| 5th moment | -12.7 | -7.9 | 7.7 | 12.2 | 8.9 |
| 6th moment | 73.3 | 59.8 | 29.8 | 52.5 | 40.5 |
| 7th moment | -200.0 | -125.5 | 86.3 | 194.3 | 132.3 |
| 8th moment | 1024.6 | 805.7 | 305.6 | 792.5 | 537.3 |
| Omega 0% | 15.37 | 17.93 | 30.40 | 31.12 | 33.30 |
| $I_1$ | 0.052% | 0.047% | 0.028% | 0.027% | 0.025% |
| $I_2$ | 0.802% | 0.845% | 0.849% | 0.826% | 0.845% |

## Panel B

| Global Minimum Risk Portfolios | | | | | |
|---|---|---|---|---|---|
| Statistics | Mean-Variance | Mean-VaR | Mean-adusted VaR | Mean-D.Dev. 0% | Omega 0% |
| **Portfolio 5 : Global CSFB** | | | | | |
| Mean | 0.70% | 0.74% | 0.73% | 0.74% | 0.74% |
| Standard Dev. | 0.61% | 0.62% | 0.66% | 0.68% | 0.67% |
| Skewness | -0.1 | -0.1 | 0.2 | 0.2 | 0.2 |
| Kurtosis | 2.9 | 2.8 | 2.5 | 2.3 | 2.4 |
| 5th moment | -0.5 | -0.4 | 1.9 | 1.6 | 1.5 |
| 6th moment | 12.1 | 11.3 | 9.7 | 7.8 | 8.8 |
| 7th moment | -0.1 | -1.8 | 12.7 | 9.9 | 9.4 |
| 8th moment | 62.0 | 55.5 | 47.7 | 33.0 | 39.4 |
| Omega 0% | 15.20 | 17.58 | 21.72 | 21.25 | 23.28 |
| $I_1$ | 0.048% | 0.044% | 0.035% | 0.036% | 0.033% |
| $I_2$ | 0.734% | 0.769% | 0.750% | 0.762% | 0.758% |
| **Portfolio 6 : Global HFR** | | | | | |
| Mean | 0.75% | 0.80% | 0.81% | 0.82% | 0.83% |
| Standard Dev. | 0.58% | 0.60% | 0.65% | 0.68% | 0.67% |
| Skewness | -0.5 | -0.4 | 0.7 | 0.7 | 0.7 |
| Kurtosis | 5.3 | 5.1 | 3.9 | 4.5 | 4.3 |
| 5th moment | -9.5 | -7.9 | 7.3 | 11.7 | 10.5 |
| 6th moment | 57.8 | 55.4 | 29.5 | 49.1 | 43.4 |
| 7th moment | -158.9 | -140.6 | 82.4 | 180.1 | 151.8 |
| 8th moment | 791.0 | 771.1 | 300.7 | 723.2 | 594.7 |
| Omega 0% | 18.66 | 21.76 | 38.22 | 40.73 | 43.98 |
| $I_1$ | 0.041% | 0.038% | 0.021% | 0.020% | 0.019% |
| $I_2$ | 0.772% | 0.818% | 0.814% | 0.821% | 0.831% |

# Appendix 6a

**Panel A**

| Global Minimum Risk Portfolios | | | | | | |
|---|---|---|---|---|---|---|
| | Mean-D.Dev. 0% | Mean-D.Dev. 0.1% | Mean-D.Dev. 0.63% | Omega 0% | Omega 0.1% | Omega 0.63% |
| **Portfolio 1 : Global Hedge Fund Index** | | | | | | |
| Mean | 0.71% | 0.71% | 0.72% | 0.73% | 0.74% | 0.73% |
| Standard Dev. | 1.66% | 1.66% | 1.67% | 1.69% | 1.71% | 1.68% |
| Skewness | -0.1 | -0.1 | -0.1 | -0.1 | 0.0 | -0.1 |
| Kurtosis | 2.8 | 2.8 | 2.8 | 2.9 | 2.9 | 2.8 |
| 5th moment | -0.3 | -0.3 | -0.4 | -0.7 | -0.7 | -0.6 |
| 6th moment | 11.3 | 11.3 | 11.3 | 12.4 | 12.7 | 11.8 |
| 7th moment | -1.0 | -1.1 | -2.5 | -7.2 | -7.7 | -5.2 |
| 8th moment | 55.0 | 55.2 | 54.5 | 65.5 | 68.1 | 59.5 |
| Omega 0% | 2.72 | 2.34 | 1.08 | 2.77 | 2.41 | 1.10 |
| $I_1$ | 0.391% | 0.427% | 0.649% | 0.390% | 0.424% | 0.646% |
| $I_2$ | 1.061% | 0.997% | 0.700% | 1.078% | 1.022% | 0.708% |
| **Portfolio 2 : Global Fund of Funds** | | | | | | |
| Mean | 0.60% | 0.60% | 0.60% | 0.61% | 0.61% | 0.61% |
| Standard Dev. | 1.40% | 1.40% | 1.40% | 1.44% | 1.44% | 1.44% |
| Skewness | -0.1 | -0.1 | -0.1 | -0.3 | -0.3 | -0.3 |
| Kurtosis | 3.4 | 3.4 | 3.5 | 4.8 | 4.8 | 4.8 |
| 5th moment | -2.3 | -2.3 | -2.7 | -8.5 | -8.5 | -8.5 |
| 6th moment | 21.9 | 21.9 | 23.0 | 51.2 | 51.2 | 51.2 |
| 7th moment | -30.3 | -30.3 | -36.7 | -161.3 | -161.3 | -161.3 |
| 8th moment | 181.2 | 181.2 | 199.5 | 739.0 | 739.0 | 739.0 |
| Omega 0% | 2.78 | 2.31 | 0.88 | 2.82 | 2.34 | 0.89 |
| $I_1$ | 0.314% | 0.351% | 0.593% | 0.310% | 0.347% | 0.589% |
| $I_2$ | 0.873% | 0.810% | 0.524% | 0.874% | 0.811% | 0.526% |

# Appendix 6b

**Panel B**

| Global Minimum Risk Portfolios | Mean-D.Dev. 0% | Mean-D.Dev. 0.1% | Mean-D.Dev. 0.63% | Omega 0% | Omega 0.1% | Omega 0.63% |
|---|---|---|---|---|---|---|
| **Portfolio 3 : Hedge Funds CSFB** | | | | | | |
| Mean | 0.76% | 0.76% | 0.76% | 0.76% | 0.77% | 0.79% |
| Standard Dev. | 0.70% | 0.70% | 0.70% | 0.69% | 0.69% | 0.69% |
| Skewness | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.0 |
| Kurtosis | 2.4 | 2.4 | 2.4 | 2.6 | 2.7 | 3.0 |
| 5th moment | 1.9 | 1.9 | 1.9 | 2.0 | 1.4 | 0.3 |
| 6th moment | 8.9 | 8.9 | 9.4 | 10.6 | 11.2 | 12.9 |
| 7th moment | 13.6 | 13.6 | 14.1 | 15.4 | 10.6 | 3.7 |
| 8th moment | 44.4 | 44.4 | 48.4 | 58.0 | 58.5 | 66.1 |
| Omega | 19.50 | 11.47 | 1.50 | 21.60 | 13.32 | 1.70 |
| $I_1$ | 0.040% | 0.062% | 0.235% | 0.036% | 0.053% | 0.207% |
| $I_2$ | 0.785% | 0.706% | 0.351% | 0.780% | 0.707% | 0.352% |
| **Portfolio 4 : Hedge Funds HFR** | | | | | | |
| Mean | 0.82% | 0.84% | 0.87% | 0.84% | 0.84% | 0.89% |
| Standard Dev. | 0.71% | 0.70% | 0.70% | 0.69% | 0.69% | 0.69% |
| Skewness | 0.7 | 0.7 | 0.5 | 0.6 | 0.6 | 0.2 |
| Kurtosis | 4.6 | 4.3 | 3.9 | 4.2 | 4.5 | 3.8 |
| 5th moment | 12.2 | 10.1 | 7.4 | 8.9 | 10.5 | 3.1 |
| 6th moment | 52.5 | 42.6 | 32.1 | 40.5 | 48.8 | 25.2 |
| 7th moment | 194.3 | 144.4 | 95.2 | 132.3 | 171.9 | 38.8 |
| 8th moment | 792.5 | 566.9 | 361.1 | 537.3 | 716.9 | 219.9 |
| Omega | 31.12 | 20.25 | 2.29 | 33.30 | 20.41 | 2.47 |
| $I_1$ | 0.027% | 0.037% | 0.173% | 0.025% | 0.037% | 0.165% |
| $I_2$ | 0.826% | 0.757% | 0.395% | 0.845% | 0.756% | 0.406% |

**Panel C**

| Global Minimum Risk Portfolios | Mean-D.Dev. 0% | Mean-D.Dev. 0.1% | Mean-D.Dev. 0.63% | Omega 0% | Omega 0.1% | Omega 0.63% |
|---|---|---|---|---|---|---|
| **Portfolio 5 : Global CSFB** | | | | | | |
| Mean | 0.74% | 0.74% | 0.75% | 0.74% | 0.74% | 0.77% |
| Standard Dev. | 0.68% | 0.67% | 0.64% | 0.67% | 0.65% | 0.66% |
| Skewness | 0.2 | 0.2 | 0.1 | 0.2 | 0.2 | 0.1 |
| Kurtosis | 2.3 | 2.3 | 2.7 | 2.4 | 2.6 | 2.9 |
| 5th moment | 1.6 | 1.6 | 1.3 | 1.5 | 1.2 | 1.1 |
| 6th moment | 7.8 | 8.2 | 11.3 | 8.8 | 9.9 | 12.0 |
| 7th moment | 9.9 | 10.1 | 10.3 | 9.4 | 7.7 | 8.5 |
| 8th moment | 33.0 | 36.2 | 57.5 | 39.4 | 46.1 | 59.7 |
| Omega | 21.25 | 13.08 | 1.51 | 23.28 | 14.62 | 1.62 |
| $I_1$ | 0.036% | 0.052% | 0.211% | 0.033% | 0.046% | 0.204% |
| $I_2$ | 0.762% | 0.678% | 0.318% | 0.758% | 0.672% | 0.330% |
| **Portfolio 6 : Global HFR** | | | | | | |
| Mean | 0.82% | 0.83% | 0.85% | 0.83% | 0.83% | 0.89% |
| Standard Dev. | 0.68% | 0.67% | 0.66% | 0.67% | 0.66% | 0.67% |
| Skewness | 0.7 | 0.7 | 0.5 | 0.7 | 0.6 | 0.1 |
| Kurtosis | 4.5 | 4.4 | 3.6 | 4.3 | 3.9 | 3.6 |
| 5th moment | 11.7 | 11.0 | 5.4 | 10.5 | 7.1 | 0.5 |
| 6th moment | 49.1 | 45.7 | 23.4 | 43.4 | 30.7 | 22.4 |
| 7th moment | 180.1 | 163.3 | 56.3 | 151.8 | 87.9 | 3.4 |
| 8th moment | 723.2 | 646.2 | 208.2 | 594.7 | 336.3 | 179.6 |
| Omega | 40.73 | 23.93 | 2.25 | 43.98 | 27.71 | 2.55 |
| $I_1$ | 0.020% | 0.031% | 0.163% | 0.019% | 0.027% | 0.156% |
| $I_2$ | 0.821% | 0.742% | 0.367% | 0.831% | 0.740% | 0.398% |